

FILM THICKNESSES IN LUBRICATED HERTZIAN CONTACTS (EHL). PART 1: TWO-DIMENSIONAL CONTACTS (LINE CONTACTS)

1. NOTATION AND UNITS

The International System of Units (SI) is used in this Item. This is a coherent system. It should be noted that any other coherent system of units can be used in place of SI where the data are given in non-dimensional form.

A	viscosity parameter = $\left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2}$	–
B	elasticity parameter = $\left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2}$	–
E_1, E_2	moduli of elasticity for materials at surfaces in contact	N m^{-2}
E'	effective modulus defined by $\frac{1}{E'} = \frac{1}{2} \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right)$	N m^{-2}
h	minimum lubricant film thickness	m
\bar{h}	non-dimensional lubricant film thickness = $\frac{h W_s}{\eta_0 u R}$	–
k	thermal conductivity of lubricant	$\text{W m}^{-1} \text{K}^{-1}$
p	pressure of lubricant at contact	N m^{-2}
R_1, R_2	radii of curvature of each of two bodies at contact point, taken as positive for convex surfaces and negative for concave surfaces	m
R	effective radius of curvature at contact, defined by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (see Section 4.1)	m
R_{q1}, R_{q2}	RMS (root mean square) roughness of contacting surfaces	m
R_{qt}	combined surface roughness of mating surfaces = $(R_{q1}^2 + R_{q2}^2)^{1/2}$	m
u_1, u_2	velocities of surfaces relative to contact point	m s^{-1}
u'_1, u'_2	velocities of surfaces tangential to surfaces	m s^{-1}
u	lubricant entraining velocity = $(u_1 + u_2)/2$ (see Section 4.2)	m s^{-1}

u'_c	velocity of contact tangential to surfaces	m s^{-1}
v	linear velocity of body	m s^{-1}
W_s	load applied per unit width of contact (see Section 4.3)	N m^{-1}
X	parameter for estimating effect of inlet shear heating (see Section 5.1.2)	–
y	perpendicular distance from contact tangent to centre of rotation of body	m
α	pressure exponent of lubricant viscosity	$\text{m}^2 \text{N}^{-1}$
γ	angle between velocity of body and contact tangent	rad
η	dynamic viscosity	N s m^{-2}
η_0	lubricant dynamic viscosity at atmospheric pressure and at operating temperature (see Section 4.5)	N s m^{-2}
θ	absolute temperature	K
λ	specific film thickness (see Section 6)	–
ν_1, ν_2	Poisson's ratio for materials at surfaces in contact	–
ρ_0	density of lubricant at atmospheric pressure and at operating temperature	kg m^{-3}
ω	angular velocity of body	rad s^{-1}

2. INTRODUCTION

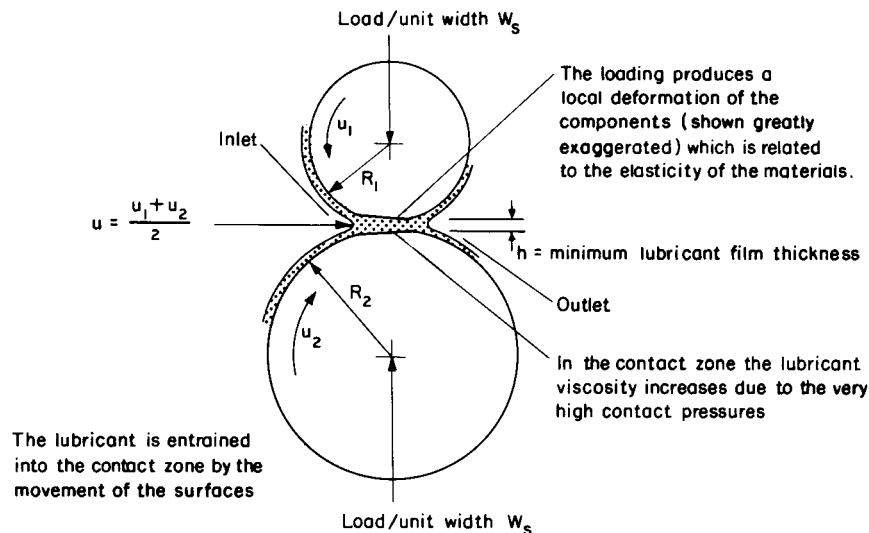
This Item provides charts and calculation methods for determining the minimum lubricant film thickness in concentrated contacts that occur, for example, in rolling bearings and gears; in such contacts, often called Hertzian contacts, the nominal contact area increases with applied load.

For the purpose of calculating the lubricant film thickness, Hertzian contacts are regarded as either two-dimensional, with nominally line contact, or three-dimensional, with nominally point contact. Nominally line contact occurs in, for example, roller bearings, roller followers on cams and spur gears while three-dimensional contacts occur in, for example, ball bearings. The charts and calculation methods presented in this Item apply to two-dimensional contacts. It is frequently possible to obtain guidance on some three-dimensional contacts using these methods (see Section 4).

The text of this Item is divided into three main parts. The first part (Section 3) describes the operation of a lubricated Hertzian contact. The second part (Sections 4 and 5) gives guidance on the specification of the design parameters, identification of the regime of operation and determination of the film thickness. The final part gives guidance on ensuring that the contact will operate with an adequate margin of safety.

3. DESCRIPTION OF CONTACT PHENOMENA

The formation of a lubricated Hertzian contact depends upon a viscous lubricant being drawn into the contact zone by the movement of the surfaces as shown in Sketch 3.1. In a typical contact the fluid pressures generated within the entrained lubricant can be a thousand times greater than those occurring in hydrodynamic journal and thrust bearings and, as a consequence of these high pressures, changes in the viscosity of the lubricant and elastic deformation of the components in contact may both be highly significant.



Sketch 3.1 A typical lubricated Hertzian contact

Normally, the lubricant film is of a uniform thickness over most of the contact but with a minimum thickness (typically 80 per cent of the uniform thickness) near the outlet of the contact zone. A knowledge of this film thickness, which can be two orders of magnitude smaller than those occurring in hydrodynamic journal and thrust bearings, is required in order to assess the likelihood of wear and surface distress.

The procedure to be followed when analysing a two-dimensional contact is shown in the flowchart of Figure 1.

4. SPECIFICATION OF PROBLEM

Certain basic information regarding the intended application is required for the design and analysis of a two-dimensional concentrated contact.

In many practical concentrated contacts the surfaces interact not at a line contact but at an elliptical contact. Provided the major axis of this elliptical contact is perpendicular to the direction of motion of the surfaces and is much larger than the minor axis, an approximate indication of the lubricant film thickness may be obtained by treating the contacting surfaces as cylindrical and applying the two-dimensional contact analysis method of this Item. The method should not be applied when the entraining direction is along the major axis of the contact ellipse.

4.1 Effective Radius of Curvature

The radius of curvature of the two cylindrical surfaces of the contacting components at the contact point must be specified. Note that a concave surface will have a negative radius of curvature. After these values have been specified, the effective radius of curvature at the contact, R , should be calculated using

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \quad (4.1)$$

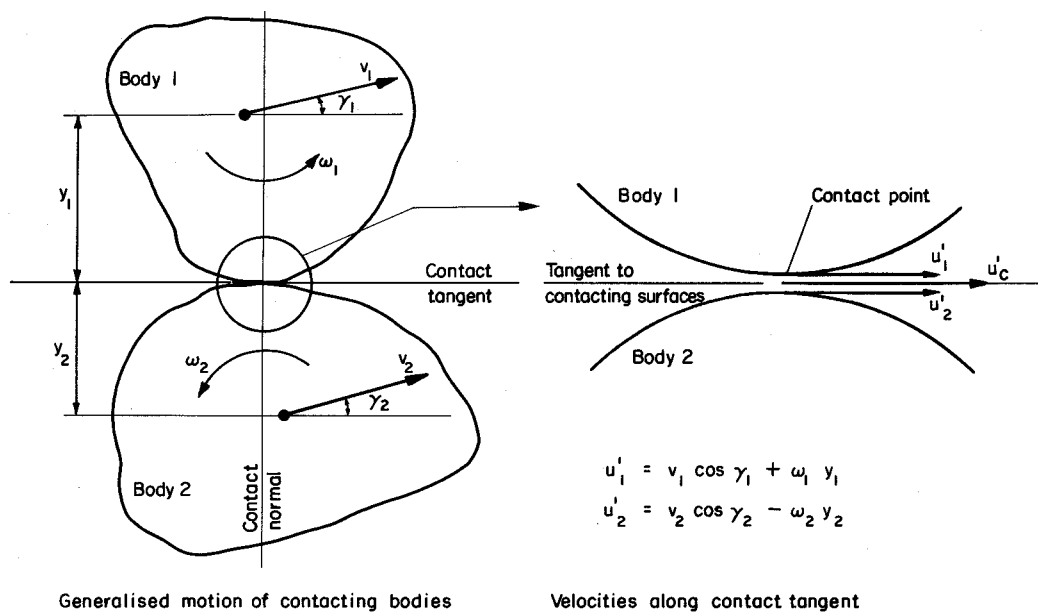
Note that where a roller contacts a flat surface the effective radius is simply equal to the radius of the roller.

4.2 Lubricant Entraining Velocity

The velocities of the two contacting surfaces, relative to the contact point, must be specified. The lubricant entraining velocity, u , is then equal to the arithmetic mean of these two values, or

$$u = \left| \frac{u_1 + u_2}{2} \right|. \quad (4.2)$$

The velocities of the two contacting surfaces should be specified in the direction of the contact tangent as illustrated in Sketch 4.1.



Sketch 4.1 Resolution of surface velocities

Care is required where the contact point itself is moving. In such cases it is often convenient to choose a co-ordinate system in which the contact point is stationary (see Examples 1 and 3 in Sections 9.1 and 9.3). Where this is not convenient, the velocity component of the contact point along the tangent, u'_c , should be specified (see Example 2 in Section 9.2). The velocities of the two contacting surfaces relative to the contact point are then given by,

$$u_1 = u'_1 - u'_c$$

and

$$u_2 = u'_2 - u'_c.$$

Example 2 of Section 9.2 illustrates the specification of velocities and the calculation of the entraining velocity for an application where the contact point moves.

Note that in pure rolling motion (motion without sliding) $u_1' = u_2'$. Sliding has little direct effect on the film thickness.

4.3 Load

For two-dimensional contacts, the load per unit width of the contact, W_s , (measured perpendicular to the direction of motion) must be specified. Where a high aspect ratio elliptical contact is being analysed, an equivalent W_s should be established based on 70 per cent of the length of the major axis of the ellipse.

4.4 Effective Modulus

The moduli of elasticity, E_1 and E_2 , and the Poisson's ratios, ν_1 and ν_2 , for the contacting materials must be specified (see Reference 1). The effective modulus for the contact can be calculated using

$$\frac{1}{E'} = \frac{1}{2} \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right). \quad (4.3)$$

Note that where similar materials contact, the effective modulus is simply

$$E' = \frac{E}{1 - \nu^2}. \quad (4.4)$$

Values of the effective modulus, $E/(1 - \nu^2)$, for typical materials are given in Table 10.1. Also given in this table are values of $(1 - \nu^2)/E$ for substitution into Equation (4.3).

4.5 Lubricant Properties

The lubricant is sometimes specified to suit the design requirements of another component. Where the designer has a free choice of lubricant, film thickness considerations may determine the choice.

For the purposes of analysing the performance of the elastohydrodynamic contact it is necessary to specify the viscosity of the lubricant at the entry to the contact, η_0 , and the pressure exponent, α , of the lubricant[†]. The viscosity, η_0 , which can be determined from a viscosity/temperature curve for the intended lubricant, needs to be carefully considered. It should be determined for a pressure equal to the ambient pressure (not the pressure of the lubricant within the contact) and a temperature representative of the expected operating surface temperature of the contacting parts. In some high speed applications the surface temperature can be appreciably higher than the bulk lubricant temperature and in these applications an allowance should be made for this difference. Values of η_0 for some lubricants are given in Table 10.2.

For many lubricants the pressure exponent of viscosity does not vary greatly with temperature and pressure for a given lubricant. However, oils are proprietary materials and the manufacturers often cannot specify a value of α nor guarantee to hold it constant. Values of α for some lubricants are given in Table 10.2.

[†] The pressure exponent of lubricant viscosity is defined by the expression $\eta = \eta_0 \exp(\alpha p)$.

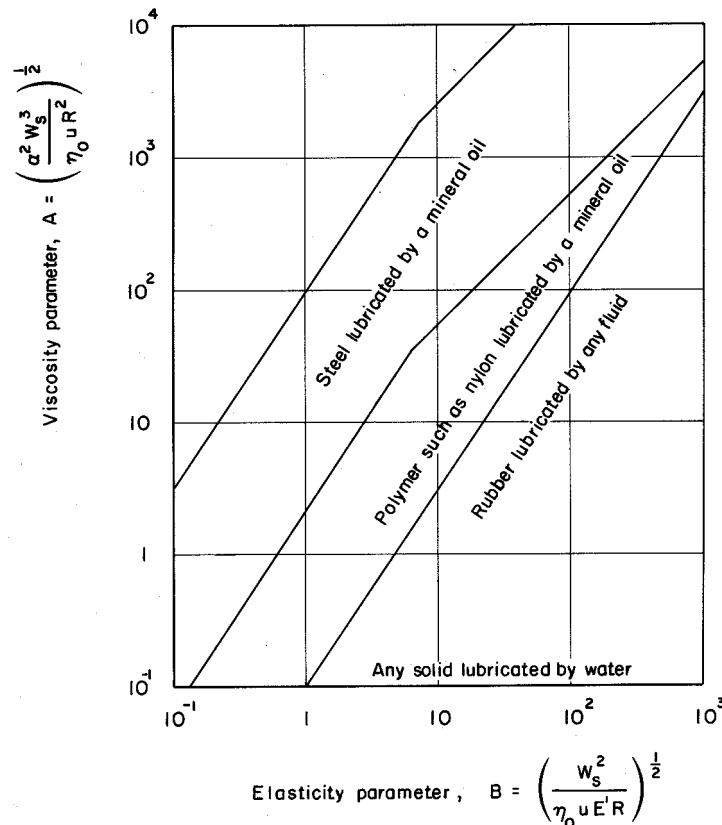
5. DETERMINATION OF LUBRICANT FILM THICKNESS

In order to determine the lubricant film thickness it is first necessary to calculate the values of the viscosity parameter[†], A, and the elasticity parameter[†], B.

$$A = \left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2} \quad (5.1)$$

$$B = \left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2} \quad (5.2)$$

The possible values of these two parameters are limited by the practical range of the operating conditions specified in Section 4 and values of A and B falling outside the regions indicated on Sketch 5.1 are extremely rare.



Sketch 5.1 Practical operating regions

Film thickness data are presented in Figure 2 as non-dimensional film thickness, \bar{h} , contours on rectangular axes of the viscosity parameter, A, and the elasticity parameter, B. The value of \bar{h} corresponding to the calculated values of A and B should be determined and the actual film thickness calculated using Equation (5.3).

[†] The viscosity parameter, A, and the elasticity parameter, B, are non-dimensional groups used to characterise a lubricated Hertzian contact. Other pairs of non-dimensional groups may be used to characterise the contact (see Derivation 12) but A and B (often referred to as g_1 and g_3 in the literature) have been chosen as the pair best suited to the presentation of film thickness data in this Item.

$$h = \frac{\bar{h}\eta_0 u R}{W_s}. \quad (5.3)$$

The adequacy of this calculated value of film thickness should be confirmed using the guidance given in Section 6. Where its value is considered inadequate it may be necessary to modify the contact (where this is possible) using the guidance given in Section 7.

5.1 Further Factors Influencing the Film Thickness

The lubricant film thickness estimate given by this Item assumes Newtonian lubricants, that is lubricants where viscosity is independent of shear rate, and isothermal conditions, where no significant inlet shear heating occurs. Significant reduction of the film thickness may occur where these two assumptions are not satisfied.

5.1.1 Non-Newtonian lubricants

In a typical contact the lubricant will be subjected to maximum shear rates of the order of 10^6 to 10^7 s^{-1} in the inlet region. The film thickness will be lower than the calculated value if the lubricant viscosity is significantly reduced by such shear rates (see Reference 2). In particular the increase in the viscosity of mineral oils obtained from polymer additives may vanish when the lubricant is sheared.

5.1.2 Inlet shear heating

Significant heating of a lubricant can occur due to shearing in the inlet region of contacts operating at relatively high speeds (see Reference 3). The resulting reduction in film thickness is given approximately by

$$\frac{\text{Film thickness with inlet shear heating}}{\text{Film thickness for isothermal conditions}} = (1 + 0.46 X)^{-0.45},$$

where

$$X = \frac{u^2}{k} \left(-\frac{d\eta_0}{d\theta} \right).$$

Note that $d\eta_0/d\theta$ is negative and so X is positive. The rate of change of viscosity with temperature, $d\eta_0/d\theta$, can be estimated from a chart of the viscosity/temperature characteristics of the chosen lubricant. However, for most lubricants the relationship between viscosity and temperature is highly nonlinear and it is advisable to estimate $d\eta_0/d\theta$ using the identity

$$\frac{d\eta_0}{d\theta} \equiv \eta_0 \frac{d}{d\theta} \left(\ln(\eta_0) \right).$$

The thermal conductivity of the lubricant, k , should be evaluated at ambient pressure and for mineral oils is given approximately by

$$k = \frac{(134.5 - 0.0633 \theta)}{\rho_0} \text{ W m}^{-1} \text{ K}^{-1},$$

where θ is the absolute temperature in K and ρ_0 is density in kg m^{-3} .

6. LIMITING CRITERIA

6.1 Specific Film Thickness (λ Value)

The degree of surface asperity contact in elastohydrodynamic lubrication can be expressed by the ratio of the minimum thickness of the lubricant film, h , (calculated assuming smooth surfaces) to the combined roughness of the two mating surfaces. This ratio is known as the specific film thickness or lambda ratio, λ , and is defined by

$$\lambda = \frac{\text{thickness of the lubricant film, } h}{\text{combined surface roughness, } R_{qt}}, \quad (6.1)$$

where the combined surface roughness is derived from the individual roughnesses of the two surfaces by the relationship

$$R_{qt} = (R_{q1}^2 + R_{q2}^2)^{1/2}, \quad (6.2)$$

and where R_{q1} and R_{q2} are the RMS (root mean square) roughnesses of the two surfaces. For a typical ground surface, the RMS roughness, R_q , is about 1.3 times the CLA (centre line average) roughness, R_a .

The roughness should be measured in a direction perpendicular to the lay of the surface (direction of machining or grinding) and some form of filtering process should be employed to attenuate the longer wavelengths that do not form part of the roughness texture. This may be achieved by using a sampling length or meter cut-off approximately equal to the Hertzian dimension of the contact (see References 4 and 5).

In general the initial value of the composite roughness will be greater than that of a successfully run-in component. This running-in is usually negligible for rolling bearings but can be considerable for components such as gears. Typical values of composite roughness for rolling bearings and gears are given in Tables 6.1 and 6.2.

TABLE 6.1

Typical Rolling Bearing Finishes	
Bearing Type	Composite roughness, μm
Ball	0.18
Spherical and cylindrical roller	0.36
Tapered and needle roller	0.23

TABLE 6.2

Typical Gear Tooth Finishes		
Method of finishing	Composite roughness, μm	
	Initial value	Run-in value
Milling	2.3 – 4.6	1.2 – 2.3
Hobbing or shaping	1.2 – 2.3	0.9 – 1.7
Shaving or grinding	0.7 – 1.4	0.6 – 1.2
Lapping	0.6 – 1.1	0.4 – 0.9
Honing	0.3 – 0.6	0.2 – 0.4

Where it is feasible to run-in a component gently, calculation of the λ value may be based on an estimation of the run-in composite roughness; otherwise the initial value should be used.

For typical engineering component surfaces, a λ value exceeding about 3 implies that there will only be very occasional penetration of the lubricant film by the surface asperities. As the value of λ decreases, solid-to-solid contact becomes more frequent until, at λ values of approximately 1, surface asperities will always be in contact. The load will, however, still be carried essentially by the lubricating film until λ values very much smaller than 1 are encountered. Most practical lubricated concentrated contacts involve λ values lying between 0.5 and 3.0. The importance of avoiding asperity contact becomes more important as the degree of sliding increases.

6.2 Acceptable λ Values

Many types of wear (see Reference 6) and surface fatigue can be associated with an inadequate λ value. In general, machine failure due to these types of surface damage would be avoided by designing for λ values exceeding about 3. However, in most applications it is not possible to achieve that value and the system will operate successfully with smaller λ values. The acceptable λ value depends on a number of factors which include the reliability requirement and the degree of sliding present. Contacts involving sliding, such as those between gear teeth, require larger λ values than are required in the pure rolling contacts of, for example, most rolling bearings. In all cases reliability is enhanced by increasing the λ value.

Scuffing is a particular form of wear that cannot be directly associated with an inadequate λ value (see Reference 7). The phenomenon, which is not fully understood, appears to be a thermal effect and cannot always be ameliorated by increasing the λ value.

In all applications where λ values less than 1 have been predicted the use of anti-wear or extreme pressure additives should be considered. Caution should, however, be exercised since certain highly reactive additives can shorten the fatigue life of the surfaces, particularly in rolling bearings.

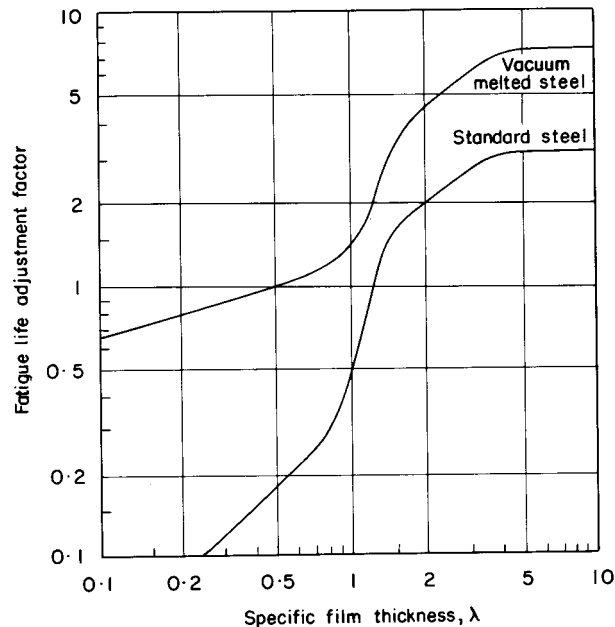
6.2.1 Acceptable λ values in gears

In gears the film thickness varies throughout the meshing cycle but the value at the pitch point is usually taken as representative of the quality of lubrication.

Values of λ exceeding 2 are unlikely to be required for successful gear operation. Experience suggests that the λ values for successful lubrication depend on the pitch line velocity and can be considerably less than 2 on low speed gears (see Reference 8).

6.2.2 Acceptable λ values in rolling bearings

Correctly selected and installed rolling bearings generally operate with λ values of around 1 which are adequate to ensure that the required basic rating life is realisable. In certain applications, however, bearing life may be extended by designing for larger λ values as shown in Sketch 6.1 (Reference 9). It should also be noted that improved performance can be obtained by using a better quality material.



Sketch 6.1 Influence of λ on rolling contact fatigue life

6.2.3 Acceptable λ values in cams

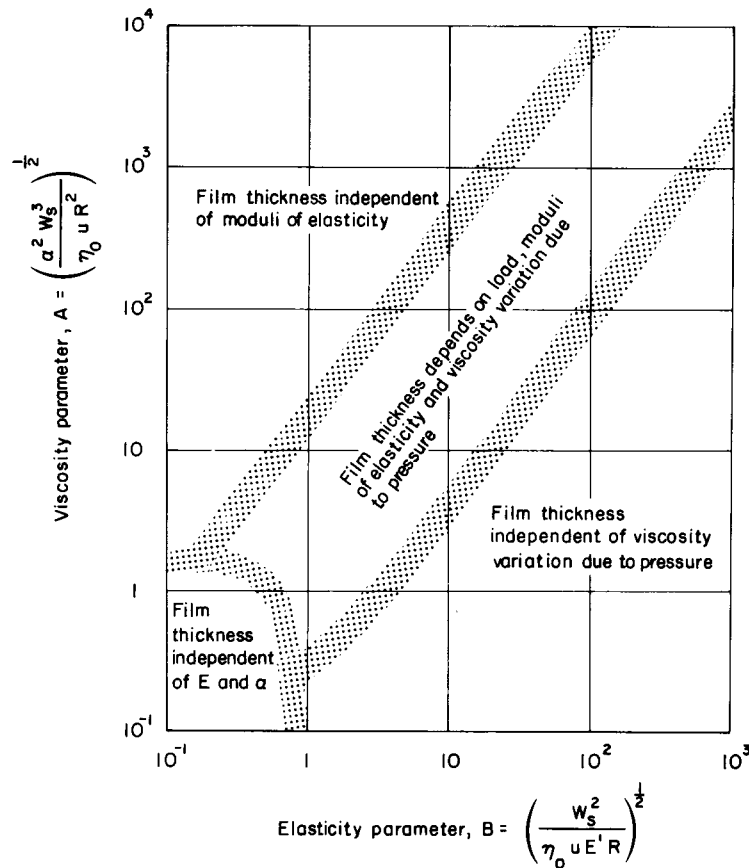
Cam/follower systems frequently operate with very small values of λ (less than 1) over certain parts of their rotational cycle. Usually these small λ values correspond to points where the entraining velocity, u , is temporarily small.

7. RE-DESIGN OF THE CONTACT

In most practical situations the load applied to the contact and the speed of operation of the mechanism incorporating the intended contact will be part of a basic design specification and will not be at the disposal of the designer. Usually, however, the design constraints will allow certain changes to be made to the geometry of the contact and to the specification of the contacting materials and the lubricant. For example, it may be possible to alter the lubricant specification, thus changing the values of η_0 and α , and to alter dimensions, such as the base circle radius of a cam, which changes the values of u and R . The effect that any of these changes will have on the film thickness depends on the regime of lubrication at the contact.

According to the values of the viscosity parameter, A, and the elasticity parameter, B, various regimes of lubrication can be identified as shown in Sketch 7.1.

With small values of both A and B (corresponding essentially to very low load situations) the film thickness is independent of both the pressure exponent of lubricant viscosity, α , and the effective modulus, E' , but is inversely proportional to the loading, W_s . Outside this region, the film thickness can be influenced by changes in α and E' as shown in the sketch but the effect of changing W_s becomes less important and has effectively no influence on film thickness in many practical applications.

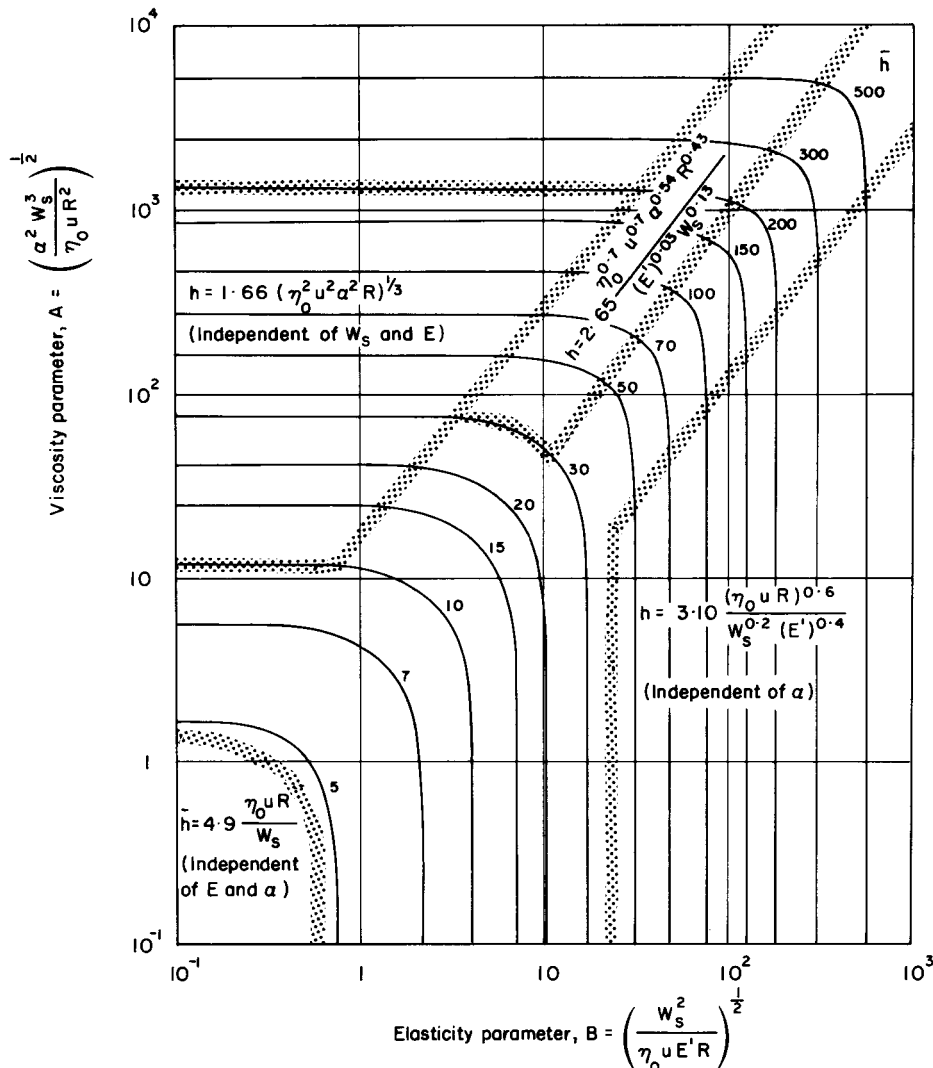


Sketch 7.1 Regimes of lubrication

The change in the lubricant film thickness produced by a change in any of the specified parameters cannot be seen directly from Figure 2 which will have been used in determining the film thickness. However, for certain ranges of values of A and B an indication of the change in the film thickness can be obtained by making use of the approximate formulae[†] given in Sketch 7.2. Caution should be exercised when changing the lubricant specification. For example, increasing the viscosity grade of the lubricant will not always give the expected increase in the film thickness since the temperature of the lubricant may increase.

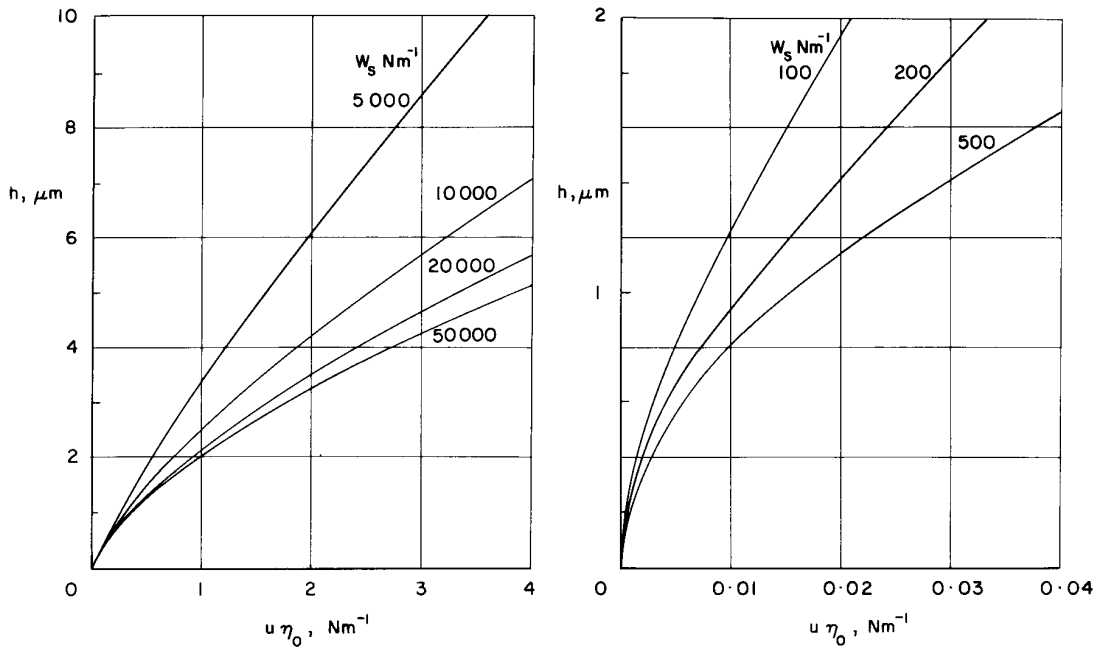
[†] Four distinct regions of operation are identified in the literature when discussing expressions for the film thickness. These four regions do not correspond exactly to the regimes shown on Sketch 7.2 but correspond to simplifying assumptions used in obtaining the film thickness expressions. The four regions are called rigid-isoviscous, rigid-variable viscosity, elastic-isoviscous and elastic-variable viscosity.

By way of illustration, the approximate variation of the film thickness, h , with W_s and the product $u\eta_0$ for two specific practical cases is shown in Sketch 7.3. Sketch 7.3a applies to the conditions that might be met in cams, while Sketch 7.3b applies to the conditions found in low speed reciprocating rubber seals on hydraulic rams.



Sketch 7.2 Approximate formulae for film thickness

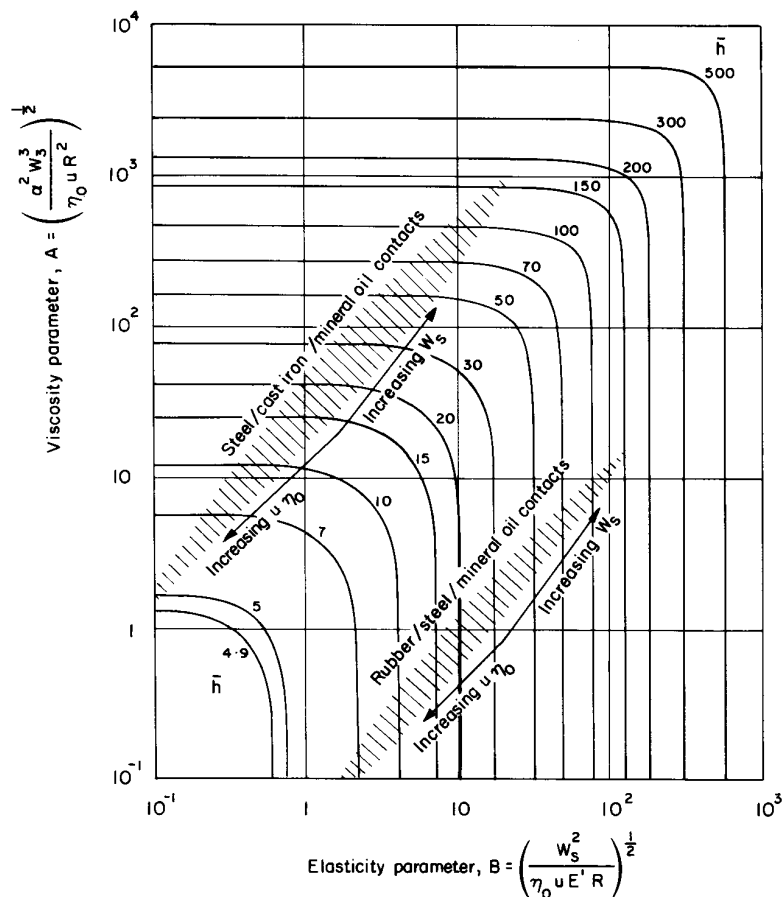
The shaded areas on Sketch 7.4 show that the range of conditions that might be met in each of these two applications corresponds to only a very limited range of values of the parameters A and B. In both cases the effect of increasing W_s is to increase both A and B while the effect of increasing $u\eta_0$ is to decrease both A and B and increase the actual film thickness, h .



a. Cast iron cam, steel roller follower and ISO VG 320 mineral oil

b. Rubber seal, steel shaft and ISO VG 100 mineral oil

Sketch 7.3 Variation of actual film thickness, h , with W_s and $u\eta_0$



Sketch 7.4 Range of A and B for a cam and for a rubber seal

8. REFERENCES AND DERIVATION

8.1 References

The references given are recommended sources of information supplementary to that in this Item

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8.2 Derivation

The derivation lists selected sources that have assisted in the preparation of this Item

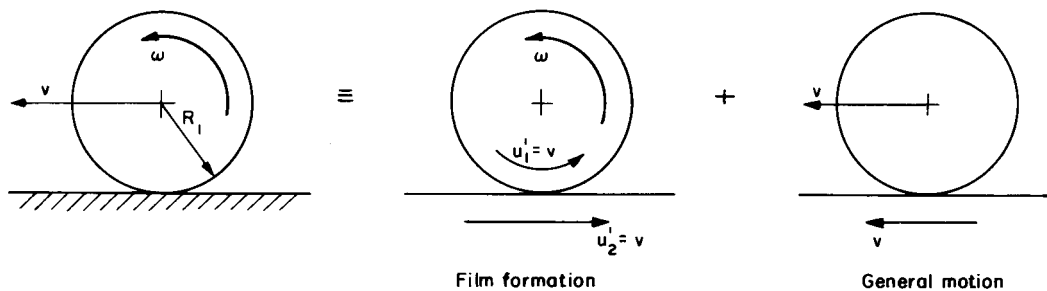
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9. EXAMPLES

9.1 Example 1

Note: This example has been devised simply to illustrate the calculation of film thickness. It is not intended to illustrate a practical application in contrast to Examples 9.2 and 9.3.

It is required to determine the minimum lubricant film thickness at the contact between a steel cylinder, of radius $R_1 = 0.02$ m and length $l = 0.05$ m, and a flat steel surface. The cylinder carries a load, W , of 2×10^4 N and rolls without slip at a linear velocity of $v = 2.3$ m s⁻¹ with an ISO VG 320 mineral oil lubricant. The motion of the cylinder is shown in Sketch 9.1.



Sketch 9.1 Cylinder rolling without slip on flat surface

The procedure to be followed when analysing the contact is shown in the flowchart of Figure 1.

9.1.1 Specification of problem

Guidance on the specification of the problem is given in Section 4.

The effective radius, R , (see Section 4.1) is equal to the radius of the cylinder.

$$R = R_1 = 0.02 \text{ m.}$$

Sketch 9.1 shows the velocities of the cylinder and the flat surface in co-ordinate axes that translate with the cylinder. The contact point is, of course, stationary in these axes and so the lubricant entraining velocity, u , (see Section 4.2) is

$$u = \frac{u_1 + u_2}{2} = \frac{v + v}{2} = v = 2.3 \text{ m s}^{-1}.$$

The load per unit width of the contact is

$$W_s = \frac{W}{l} = \frac{2 \times 10^4}{0.05} = 4 \times 10^5 \text{ N m}^{-1}.$$

The surface temperature of the cylinder and the flat surface is estimated to be 50°C at which temperature the lubricant properties (see Section 4.5) are,

$$\eta_0 = 0.15 \text{ N s m}^{-2}$$

$$\alpha = 22 \times 10^{-9} \text{ m}^2 \text{ N}^{-1}.$$

9.1.2 Determination of lubricant film thickness

Guidance on determining the lubricant film thickness is given in Section 5.

The viscosity parameter and the elasticity parameter are calculated to be

$$A = \left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2} = \left(\frac{(22 \times 10^{-9})^2 \times (4 \times 10^5)^3}{0.15 \times 2.3 \times (0.02)^2} \right)^{1/2} = 474$$

and

$$B = \left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2} = \left(\frac{(4 \times 10^5)^2}{0.15 \times 2.3 \times 0.23 \times 10^{12} \times 0.02} \right)^{1/2} = 10.0.$$

From Figure 2, the non-dimensional film thickness, \bar{h} , is 100. The actual film thickness is, therefore,

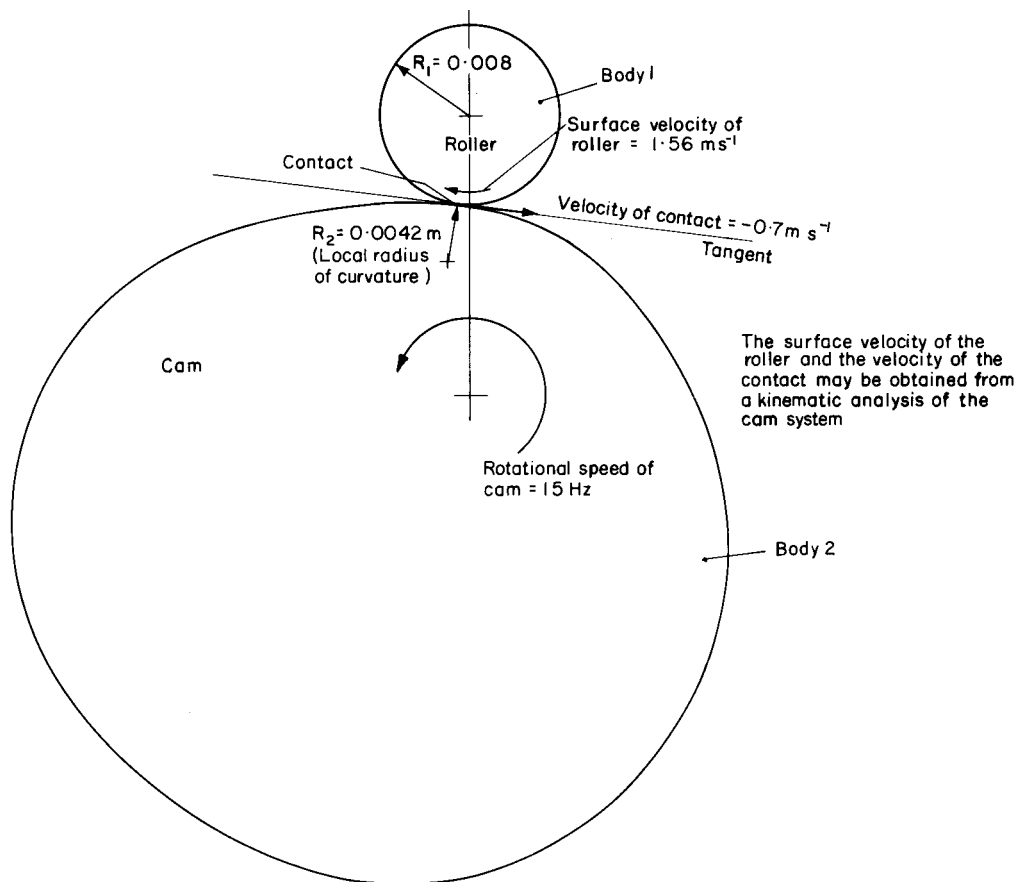
$$h = \frac{\bar{h} \eta_0 u R}{W_s} = \frac{100 \times 0.15 \times 2.3 \times 0.02}{4 \times 10^5} = 1.7 \text{ } \mu\text{m}.$$

9.2 Example 2

It is required to check the lubricant film thickness at the contact between a disc cam and a translating cylindrical roller follower with the following geometry and operating conditions:

base circle radius	0.016 m
cam/follower effective width	0.011 m
roller follower radius	0.008 m
rotational speed of cam	15 Hz
lubricant	ISO VG 320
operating temperature	60 °C
material of cam	chill cast iron
material of follower	bearing steel
surface roughness of cam	0.38 μm RMS
surface roughness of follower	0.15 μm RMS.

The designer determines that a possible critical condition exists at a point on the return segment of the cam shortly before the follower returns to the zero lift position. The designer estimates that the conditions at this point are as shown in Sketch 9.2. The contact load at this point is 50 N.



Sketch 9.2 Radii and velocities at the contact

9.2.1 Specification of problem

The designer refers to Section 4 and specifies the problem.

The effective radius of curvature at the contact is given by (see Equation (4.1))

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.0042} + \frac{1}{0.008},$$

that is $R = 0.00275$ m.

Negligible slip is assumed at the contact and the velocities of the contacting surfaces along the tangent are $u'_1 = u'_2 = 1.56 \text{ m s}^{-1}$. The velocity of the contact point, measured in the same sense as the surface velocities, is $u'_c = -0.7 \text{ m s}^{-1}$.

The velocities of the contacting surfaces relative to the contact (see Section 4.2) are given by,

$$u_1 = u_2 = u'_1 - u'_c = u'_2 - u'_c = 1.56 - (-0.7) = 2.26 \text{ m s}^{-1}.$$

The entraining velocity, u , (see Equation (4.2)) is therefore 2.26 m s^{-1} .

The load per unit width of the contact is given by, $W_s = 50/0.011 = 4545 \text{ N m}^{-1}$.

The effective modulus, E' , is given by (see Equation (4.3)),

$$\frac{1}{E'} = \frac{1}{2} \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right).$$

Values of $4.5 \times 10^{-12} \text{ m}^2 \text{ N}^{-1}$ and $5.5 \times 10^{-12} \text{ m}^2 \text{ N}^{-1}$ for the factor $(1 - \nu^2)/E$ are selected from Table 10.1 and hence

$$\frac{1}{E'} = \frac{1}{2} (4.5 + 5.5) \times 10^{-12} = 5.0 \times 10^{-12} \text{ m}^2 \text{ N}^{-1}.$$

The dynamic viscosity of the lubricant, η_0 , is found to be 0.094 N s m^{-2} at a temperature of 60°C . The pressure exponent of lubricant viscosity, α , is not known but a value of $26.3 \times 10^{-9} \text{ m}^2 \text{ N}^{-1}$ is selected from Table 10.2.

Summarising, the basic parameters of the contact are

$$\begin{aligned} R &= 0.00275 \text{ m} \\ u &= 2.26 \text{ m s}^{-1} \\ W_s &= 4545 \text{ N m}^{-1} \\ E' &= 0.2 \times 10^{12} \text{ N m}^{-2} \\ \eta_0 &= 0.094 \text{ N s m}^{-2} \\ \alpha &= 26.3 \times 10^{-9} \text{ m}^2 \text{ N}^{-1}. \end{aligned}$$

9.2.2 Determination of lubricant film thickness

In order to determine the lubricant film thickness (see Section 5) the designer first calculates the viscosity parameter, A, and the elasticity parameter, B. Thus

$$\begin{aligned} A &= \left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2} = \left(\frac{(26.3 \times 10^{-9})^2 \times 4545^3}{0.094 \times 2.26 \times 0.00275^2} \right)^{1/2} = 6.36, \\ B &= \left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2} = \left(\frac{4545^2}{0.094 \times 2.26 \times 0.2 \times 10^{12} \times 0.00275} \right)^{1/2} = 0.42. \end{aligned}$$

Using Figure 2, the value of the non-dimensional lubricant film thickness, \bar{h} , corresponding to the calculated values of the parameters A and B, is 7.5. Using Equation (5.3), the film thickness is

$$h = \frac{\bar{h} \eta_0 u R}{W_s} = \frac{7.5 \times 0.094 \times 2.26 \times 0.00275}{4545} = 0.96 \text{ } \mu\text{m}.$$

9.2.3 Calculation of λ value

Referring to Section 6, the designer uses Equation (6.2) to determine the combined surface roughness, R_{qt} , that is,

$$R_{qt} = (R_{q1}^2 + R_{q2}^2)^{1/2} = (0.15^2 + 0.38^2)^{1/2} \times 10^{-6} = 0.4 \mu\text{m}.$$

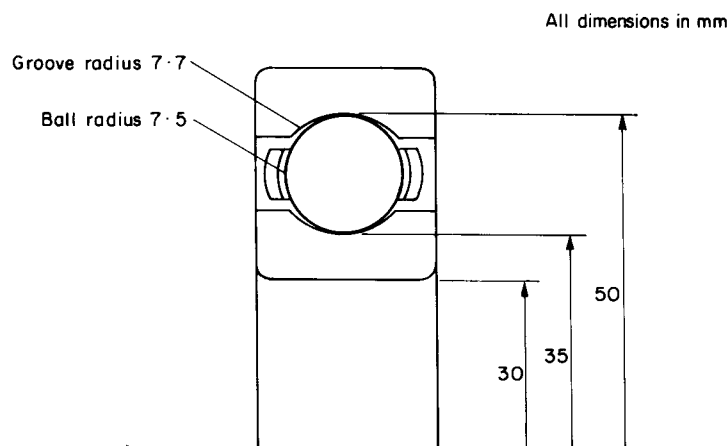
Using Equation (6.1), the specific film thickness is

$$\lambda = \frac{h}{R_{qt}} = \frac{0.96}{0.4} = 2.4.$$

This value of λ is considered acceptable for a cam with a roller follower. However, the designer would be advised to check its value for all potentially critical points on the cam. These points are likely to be where the entraining velocity, u , is small.

9.3 Example 3

It is required to determine the lubricant film thickness in a radial ball bearing. The bearing supports a pure radial load of 3000 N and rotates at 1500 rev/min. The lubricant is a VG 46 mineral oil and the bearing temperature is estimated to be 60 °C. The bearing dimensions are as shown in Sketch 9.3.

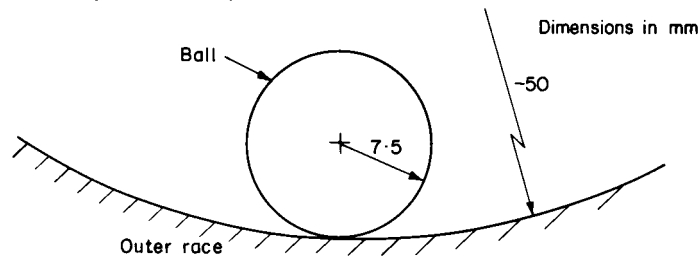


Sketch 9.3 Dimensions of radial ball bearing

In radial ball bearings each ball makes three-dimensional contact with the race grooves. The contact areas are elliptical in shape but in this example it is shown that the entraining direction is perpendicular to the major axis of the contact ellipse. The contact is then approximated to a line contact and the two-dimensional analysis of this Item is applied.

9.3.1 Specification of problem

The radii of curvature of the contacting bodies, in the plane containing the direction of motion of the rolling elements, are shown in Sketch 9.4.



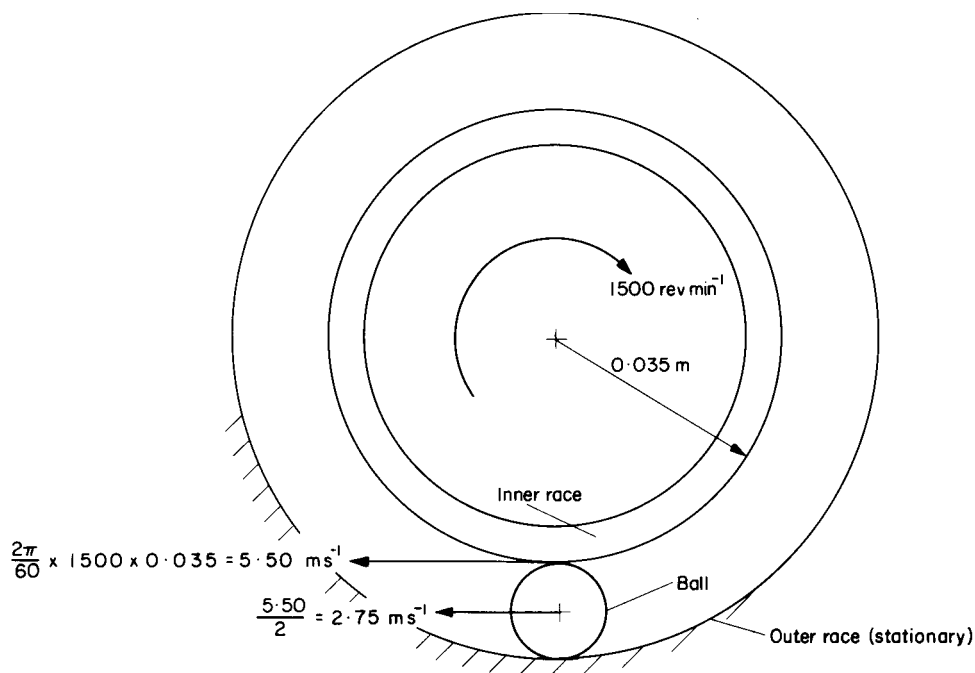
Sketch 9.4 Radii of curvature in plane of motion

Note that the outer race surface is concave and its radius of curvature has, therefore, been specified as negative. The effective radius of the contact is given by Equation (4.1), or

$$\frac{1}{R} = \frac{1}{0.0075} + \frac{1}{(-0.05)},$$

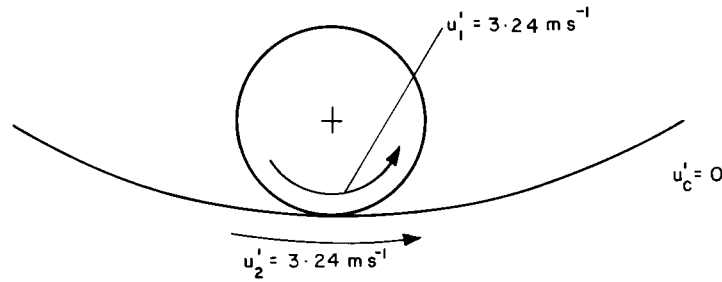
that is $R = 0.0088 \text{ m}$.

In order to determine the lubricant entraining velocity, u , it is first necessary to establish the motion of the rolling elements of the bearing. For a bearing with a stationary outer race, this motion may be established knowing the rotational speed of the bearing and the radius of the inner race as shown in Sketch 9.5.



Sketch 9.5 Instantaneous velocities of a rolling element

The ball/cage assembly has a pitch radius of 0.0425 m and rotates about the axis of the bearing with an angular velocity of $2.75/0.0425 = 64.7 \text{ rad s}^{-1}$. The velocity of the outer race surface in co-ordinate axes that are stationary with respect to the ball/cage assembly is thus $64.7 \times 0.05 = 3.24 \text{ m s}^{-1}$ as shown in Sketch 9.6.



Sketch 9.6 Surface velocities at outer contacts

The contact is stationary in these co-ordinate axes and so the lubricant entraining velocity, u , is simply

$$u = \frac{u_1 + u_2}{2} = 3.24 \text{ m s}^{-1}.$$

In a rolling bearing the rolling elements do not, in general, carry equal loads (see Reference 10). For a radial ball bearing under pure radial load the maximum load on a ball, W , due to the external load on the bearing is given approximately by

$$W = 5 \frac{F_r}{Z},$$

where F_r is the external radial load on the bearing and Z is the number of rolling elements.

For the bearing considered, $F_r = 3000 \text{ N}$ and $Z = 10$ so that

$$W = 5 \times \frac{3000}{10} = 1500 \text{ N}.$$

In addition to this load, each contact between a ball and the outer race must exert a centripetal force on each ball. For all but very high speed bearings this additional load is small and has been neglected in this example.

Where a three-dimensional contact is being approximated to a two-dimensional contact it is first necessary to estimate the dimensions of the contact ellipse. A method for calculating these contact dimensions is given in Reference 4 and the calculations for this particular example are presented in Appendix A.

It is found that the aspect ratio of the contact ellipse is approximately 9.4 with the major axis perpendicular to the direction of motion. This aspect ratio is sufficiently high to allow a two-dimensional analysis to be carried out using an equivalent W_s based on 70 per cent of the length of the major axis of the contact ellipse. Under the applied maximum load of 1500 N the length of the major axis of the contact ellipse is 0.0044 m and hence the equivalent W_s is given by

$$W_s = \frac{1500}{0.7 \times 0.0044} = 4.87 \times 10^5 \text{ N m}^{-1}.$$

It is assumed that the rolling elements and the races of the bearing are manufactured from similar materials. The effective modulus for the contact, E' , is therefore given by Equation (4.4) and a value of $220 \times 10^9 \text{ N m}^{-2}$ is selected from Table 10.1.

At the bearing temperature of 60°C the lubricant is known to have a viscosity of $\eta_0 = 17.7 \times 10^{-3} \text{ N s m}^{-2}$. A value for α is not known but a value of $20.5 \times 10^{-9} \text{ m}^2 \text{ N}^{-1}$ is selected from Table 10.2.

Summarising, the parameters of the contact are

$$\begin{aligned} R &= 0.0088 \text{ m} \\ u &= 3.24 \text{ m s}^{-1} \\ W_s &= 4.87 \times 10^5 \text{ N m}^{-1} \\ E' &= 220 \times 10^9 \text{ N m}^{-2} \\ \eta_0 &= 17.7 \times 10^{-3} \text{ N s m}^{-2} \\ \alpha &= 20.5 \times 10^{-9} \text{ m}^2 \text{ N}^{-1}. \end{aligned}$$

9.3.2 Determination of lubricant film thickness

The viscosity parameter and the elasticity parameter are

$$A = \left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2} = \left(\frac{(20.5 \times 10^{-9})^2 \times (4.87 \times 10^5)^3}{17.7 \times 10^{-3} \times 3.24 \times 0.0088^2} \right)^{1/2} = 3306,$$

and

$$B = \left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2} = \left(\frac{(4.87 \times 10^5)^2}{17.7 \times 10^{-3} \times 3.24 \times 220 \times 10^9 \times 0.0088} \right)^{1/2} = 46.2.$$

From Figure 2 the non-dimensional film thickness, \bar{h} , is 360. The actual film thickness is, therefore,

$$h = \frac{\bar{h} \eta_0 u R}{W_s} = \frac{360 \times 17.7 \times 10^{-3} \times 3.24 \times 0.0088}{4.87 \times 10^5} = 0.37 \text{ } \mu\text{m}.$$

This film thickness value is for the outer race contact with the heaviest loaded ball. The film thickness at the corresponding inner race contact will differ from this value.

9.3.3 Calculation of λ value.

From Table 6.1 in Section 6.1 a typical combined surface roughness, R_{qt} , for a ball bearing is $0.18 \text{ } \mu\text{m}$. The lambda ratio is given by,

$$\lambda = \frac{h}{R_{qt}} = \frac{0.37}{0.18} = 2.1.$$

10. TABLES

TABLE 10.1 Values of Effective Modulus and $(1-\nu^2)/E$ for Typical Materials

<i>Material</i>	<i>Tensile modulus, E</i> (10^9 N m^{-2})	<i>Poisson's ratio, ν</i>	<i>Effective modulus</i> $E/(1-\nu^2)$ (10^9 N m^{-2})	$(1-\nu^2)/E$ ($10^{-12} \text{ m}^2 \text{ N}^{-1}$)
Metals:				
Iron, grey cast	109	0.26	117	8.6
Iron, maleable cast	170	0.26	182	5.5
Iron, spheroidal-graphite	159	0.26	171	5.9
Steel, low alloy	196	0.30	215	4.6
Steel, medium and high alloy	200	0.30	220	4.6
Steel, stainless	193	0.30	212	4.7
Steel, high speed	212	0.30	233	4.3
Bronze, aluminium	117	0.33	131	7.6
Bronze, leaded	97	0.33	109	9.2
Bronze, phosphor	110	0.33	123	8.1
Brass	100	0.33	112	8.9
Aluminium	62	0.33	70	14.4
Aluminium alloy	70	0.33	79	12.7
Magnesium alloys	41	0.33	46	21.7
Zinc alloys	50	0.27	54	18.5
Polymers:				
Acetal (polyformaldehyde)	2.7	0.39	3.2	314
Nylons (polyamides)	1.9	0.4	2.3	442
Polyethylene, high density	0.9	0.35	1.0	975
Phenol formaldehyde (filled)	7.0	0.4	8.3	120
Rubber, natural (25% carbon black "mechanical" rubber)	0.004	0.5	0.005	188 000
Ceramics:-				
Alumina (Al_2O_3)	390	0.28	423	2.4
Graphite, high strength	27	0.33	30	33
Cemented carbides	450	0.19	467	2.1
Silicon carbide (SiC)	450	0.19	467	2.1
Silicon nitride (Si_3N_4)	314	0.26	337	3.0

TABLE 10.2 Dynamic Viscosity and Pressure Exponent of Some Lubricants

MINERAL OILS	Nearest ISO VG	KVI	Dynamic viscosity, η , measured at atmospheric pressure ($10^{-3} \text{ N s m}^{-2}$)			Pressure exponent, α ($10^{-9} \text{ m}^2 \text{ N}^{-1}$)		
			30 °C	60 °C	100 °C	30 °C	60 °C	100 °C
<u>High VI oils</u>								
Light machine oil	32	108	38	12.1	5.3	–	18.4	13.4
Heavy machine oil	100	96	153	34	9.1	23.7	20.5	15.8
Heavy machine oil	150	96	250	50.5	12.6	25.0	21.3	17.6
Cylinder oil	460	96	810	135	26.8	34	28	22
<u>Medium VI oils</u>								
Spindle oil	15	–	18.6	6.3	2.4	20	16	13
Light machine oil	32	68	45	12	3.9	28	20	16
Medicinal white oil	68	63	107	23.3	6.4	29.6	22.8	17.8
Heavy machine oil	75	84	122	26.3	7.3	27.0	21.6	17.5
Heavy machine oil	100	38	171	31	7.5	28	23	18
<u>Low VI oils</u>								
Spindle oil	22	–6	30.7	8.6	3.1	25.7	20.3	15.4
Heavy machine oil	100	–7	165	30.0	6.8	33.0	23.8	16.0
Heavy machine oil	150	8	310	44.2	9.4	34.6	26.3	19.5
Cylinder oil	1000	–	2000	180	24	41.5	29.4	25.0

OTHER LUBRICANTS	Dynamic viscosity, η , measured at atmospheric pressure ($10^{-3} \text{ N s m}^{-2}$)			Pressure exponent, α ($10^{-9} \text{ m}^2 \text{ N}^{-1}$)		
	30 °C	60 °C	100 °C	30 °C	60 °C	100 °C
Water	0.80	0.47	0.28	0	0	0
Ethylene oxide-propylene oxide copolymer	204	62.5	22.5	17.6	14.3	12.2
Castor oil	360	80	18.0	15.9	14.4	12.3
Di(2-ethylhexyl) phthalate	43.5	11.6	4.05	20.8	16.6	13.5
Glycerol (glycerine)	535	73	13.9	5.9	5.5	3.6
Polypropylene glycol 750	82.3	–	–	17.8	–	–
Polypropylene glycol 1500	177	–	–	17.4	–	–
Hyvis 05	91.1	–	–	29.3	–	–
Hyvis 07	211	–	–	30.2	–	–
Santotrac 40	30	–	–	34.5	–	–
Tri-arylphosphate ester	25.5	–	–	31.6	–	–
Di(2-ethylhexyl) sebacate	14.7	–	–	9.3	–	–

These tables have been compiled from published data. New data continually become available and the user should always consult the manufacturer of the particular lubricant.

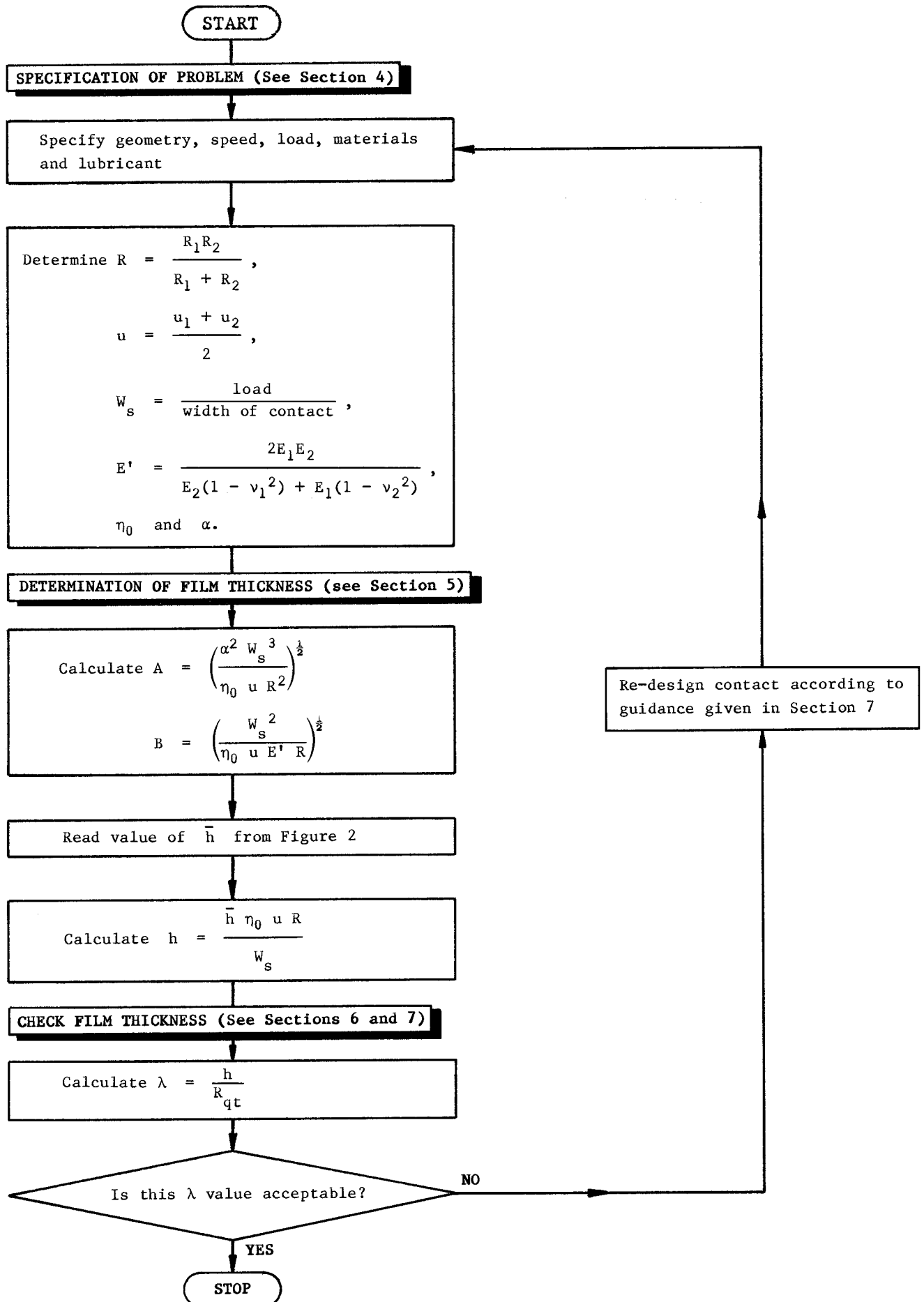


FIGURE 1 FLOWCHART FOR TWO-DIMENSIONAL CONTACT ANALYSIS

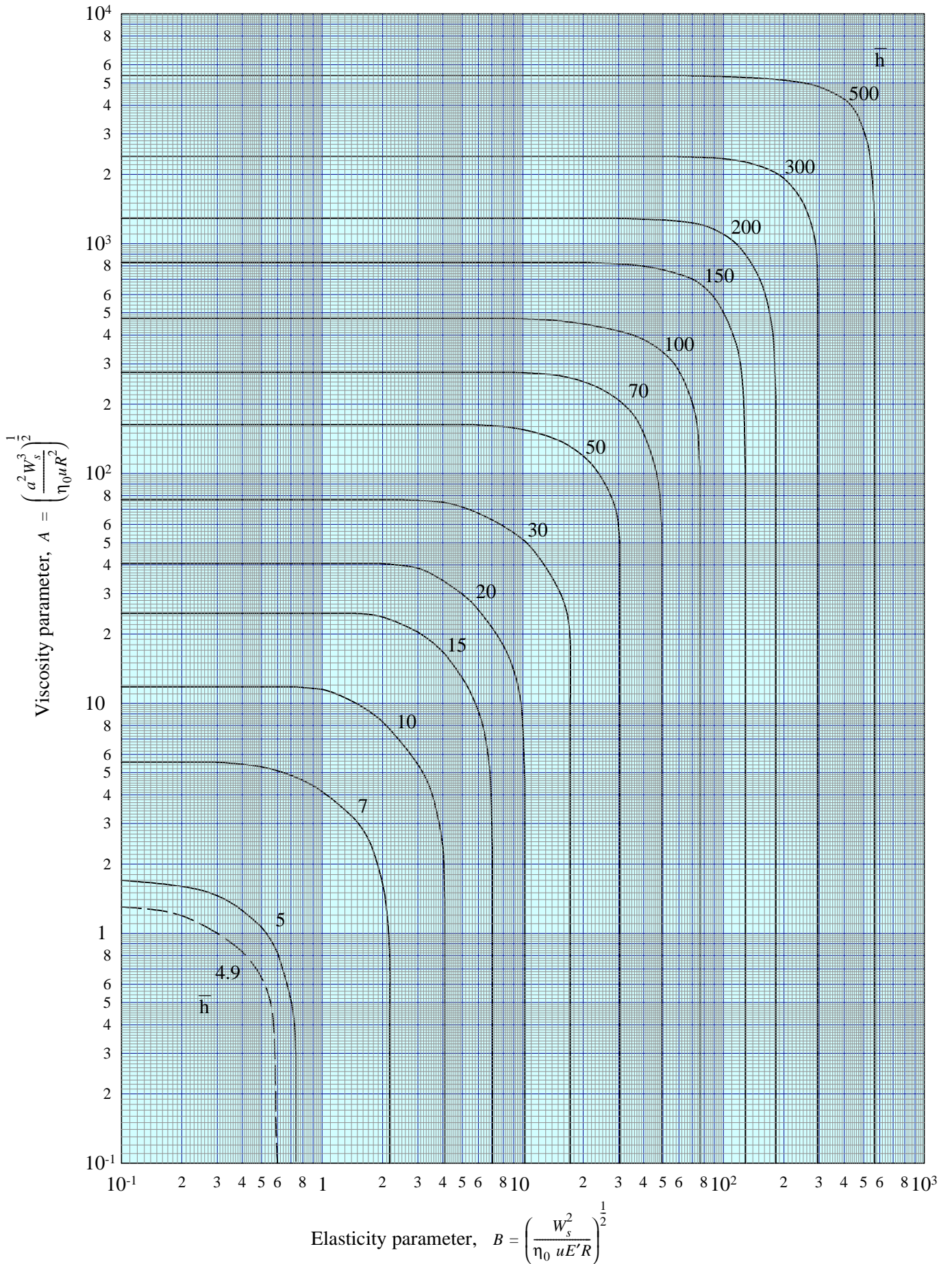


FIGURE 2 NON-DIMENSIONAL FILM THICKNESS, \bar{h}

APPENDIX A CALCULATION OF CONTACT ELLIPSE DIMENSIONS

A1. NOTATION AND UNITS

The notation given here has been based on, but is not identical to[†], that given in Reference 4, and applies only for the equations used in this Appendix.

A	body geometry parameter, one half of minor principal relative curvature	m^{-1}
a	contact ellipse major semi-axis dimension	m
B	body geometry parameter, one half of major principal relative curvature	m^{-1}
b	contact ellipse minor semi-axis dimension	m
C_a	non-dimensional coefficient associated with a	–
C_β	non-dimensional coefficient associated with β	–
E	modulus of elasticity	$N\ m^{-2}$
E'	effective modulus [†] defined by $\frac{1}{E'} = \frac{1}{2} \left(\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right)$	$N\ m^{-2}$
P	total normal applied load	N
R	principal radius of curvature of body	m
W	calculation parameter given by Equation (A3.5)	–
β	ellipse semi-axes ratio, given by b/a	–
ν	Poisson's ratio [†]	–
ω	angle between the planes in which principal curvatures lie	–

Subscripts

1, 2 where only one subscript is given it refers to body 1 or body 2 respectively; where two subscripts are given the second subscript indicates the plane of curvature.

A2. NOTES

This Appendix presents the calculation of the contact ellipse dimensions required in Example 3 of Section 9.3. The method of calculation follows that presented in Reference 4 which assumes unlubricated contacts. Contact dimensions calculated by this, or an equivalent, method are those normally used when assessing an elasto-hydrodynamically lubricated contact.

[†] The notation given in Reference 4 used the symbol σ for Poisson's ratio and used a material constant, $k = (1 - \nu^2) / \pi E$, in place of E' .

A3. CALCULATIONS

In order to determine the contact ellipse dimensions it is first necessary to calculate the body geometry parameters A and B . A and B are constants whose values depend on the magnitude of the principal curvatures and the angle, ω , between the planes in which the principal curvatures lie. In general,

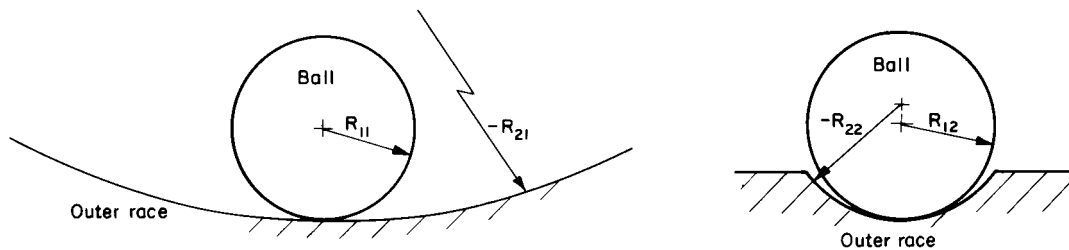
$$A = \frac{1}{4} \left\{ \frac{1}{R_{11}} + \frac{1}{R_{12}} + \frac{1}{R_{21}} + \frac{1}{R_{22}} - \left[\left(\frac{1}{R_{11}} - \frac{1}{R_{12}} \right)^2 + \left(\frac{1}{R_{21}} - \frac{1}{R_{22}} \right)^2 + 2 \left(\frac{1}{R_{11}} - \frac{1}{R_{12}} \right) \left(\frac{1}{R_{21}} - \frac{1}{R_{22}} \right) \cos 2\omega \right]^{1/2} \right\} \quad (\text{A3.1})$$

and

$$B = \frac{1}{4} \left\{ \frac{1}{R_{11}} + \frac{1}{R_{12}} + \frac{1}{R_{21}} + \frac{1}{R_{22}} + \left[\left(\frac{1}{R_{11}} - \frac{1}{R_{12}} \right)^2 + \left(\frac{1}{R_{21}} - \frac{1}{R_{22}} \right)^2 + 2 \left(\frac{1}{R_{11}} - \frac{1}{R_{12}} \right) \left(\frac{1}{R_{21}} - \frac{1}{R_{22}} \right) \cos 2\omega \right]^{1/2} \right\}, \quad (\text{A3.2})$$

where the notation for the principal planes of curvature must be chosen such that $B \geq A$.

The principal radii of curvature of the contacting body surfaces are as shown in Sketch A3.1



Sketch A3.1 Principal radii of curvature

The radii R_{21} and R_{22} refer to concave surfaces and therefore have negative values. The values of the principal radii for the ball bearing of Example 3 of Section 9.3 are

$$\begin{aligned} R_{11} &= R_{12} = 0.0075 \text{ m} \\ R_{21} &= -0.050 \text{ m} \\ R_{22} &= -0.0077 \text{ m} \\ \omega &= 90^\circ. \end{aligned}$$

With these values the expressions for A and B simplify to,

$$A = \frac{1}{2} \left(\frac{1}{R_{11}} + \frac{1}{R_{22}} \right) = \frac{1}{2} \left(\frac{1}{0.0075} + \frac{1}{(-0.0077)} \right) = 1.73$$

and

$$B = \frac{1}{2} \left(\frac{1}{R_{11}} + \frac{1}{R_{21}} \right) = \frac{1}{2} \left(\frac{1}{0.0075} + \frac{1}{(-0.050)} \right) = 56.67.$$

The contact ellipse semi-axes ratio, β , and the contact ellipse major semi-axis dimension, a , are given by,

$$\beta = C_{\beta} \left(\frac{A}{B} \right)^{2/3} \quad (\text{A3.3})$$

and
$$a = \frac{C_a W}{(A + B)} \left(\frac{A}{B} \right)^{-1/3}, \quad (\text{A3.4})$$

where
$$W = \left(\frac{3P}{2E'} \times (A + B)^2 \right)^{1/3}. \quad (\text{A3.5})$$

The non-dimensional coefficients, C_{β} and C_a , are obtained from Figure 1 of Reference 4 as $C_{\beta} = 1.09$ and $C_a = 1.20$.

From Table.10.1 a value of $220 \times 10^9 \text{ N m}^{-2}$ is selected for E' . Substitution of values into Equations (A3.3), (A3.4) and (A3.5) gives,

$$W = \left(\frac{3 \times 1500}{2 \times 220 \times 10^9} \times (1.73 + 56.67)^2 \right)^{1/3} = 0.0327$$

hence
$$\beta = 1.09 \times \left(\frac{1.73}{56.67} \right)^{2/3} = 0.106,$$

and
$$a = \frac{1.20 \times 0.0327}{(1.73 + 56.67)} \left(\frac{1.73}{56.67} \right)^{-1/3} = 0.0022 \text{ m}.$$

The contact ellipse has an aspect ratio of $1/0.106 = 9.4$ and the length of its major axis is $2 \times 0.0022 = 0.0044 \text{ m}$.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the Working Party on Elastohydrodynamic Lubrication which has the following membership:

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Prof. J. Halling – Independent

Members
Dr A. Dyson – Independent
Dr J.A. Greenwood – University of Cambridge
Dr G.M. Hamilton – University of Reading
Dr S.Y. Poon – I.T. Research Ltd
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and which was appointed by the Tribology Steering Group. The Tribology Steering Group, which also approved the final draft, first met in 1973 and now has the following membership:

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The work on this Item was carried out in the Mechanical Motion and System Dynamics Group of ESDU under the supervision of Mr C.J. Loughton. The member of staff who undertook the technical work involved in the assessment of available information and the construction and subsequent development of the Item was

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