INTRODUCTION TO THE USE OF LINEAR ELASTIC FRACTURE MECHANICS IN ESTIMATING FATIGUE CRACK GROWTH RATES AND RESIDUAL STRENGTH OF COMPONENTS

1. INTRODUCTION

This Item introduces the concepts of linear elastic fracture mechanics (LEFM) and shows how these concepts may be used for analysing the behaviour of cracked structures under static or fatigue loadings. The concepts provide an analytical method based upon the stress intensity factor, which characterises the stress distribution in the vicinity of the crack tip, and are valid in design applications provided that gross yielding does not occur. Linear elastic fracture mechanics can be used to describe ultimate static failure of low toughness high strength materials used in aerospace and other specialised applications. It should not be used to describe static failure of high toughness low strength materials (for example, materials used in the construction of pressure vessels). Under fatigue loading, crack growth rates can be correlated by the stress intensity factor for a wide range of materials, because even in the more ductile materials the amount of plastic flow that can occur under fatigue loading is restricted. It should not be used in cases where the fatigue cycles involve extensive plastic deformation, often referred to as “low cycle” fatigue.

This Item describes the basic equations of the linear elastic fracture mechanics analysis and the limitations of usage. Many of the data required in the application of these methods are provided in ESDU Data Items. They may be located via the current ESDU Index.

2. NOTATION

a	crack half-length for a crack free to extend at both ends, or crack length for a crack free to extend at one end only (see Section 3.1)	m	in
a_c	critical crack length at failure	m	in
a_0	initial crack length	m	in
a_1	effective crack length	m	in
$2b$	characteristic dimension of component width	m	in
C	coefficient in simple crack growth rate law defined by $da/dN = C(\Delta K)^n$	$m(m^{3/2}/N)^n$	$m(m^{3/2}/lbf)^n$
E	modulus of elasticity	N/m^2	lbf/in^2
G	release rate of potential energy per unit area	N/m	lbf/in
H	effective modulus of elasticity (see Section 5.1.2)	N/m^2	lbf/in^2
K	stress intensity factor (Mode I)	$N/m^{3/2}$	$lbf/in^{3/2}$
K_c	critical value of stress intensity factor (also plane stress fracture toughness)	$N/m^{3/2}$	$lbf/in^{3/2}$

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K_{cl}	stress intensity factor value at crack closure	$N/m^{3/2}$	$lbf/in^{3/2}$
K_p	stress intensity factor corrected for plasticity	$N/m^{3/2}$	$lbf/in^{3/2}$
K_R	crack growth resistance	$N/m^{3/2}$	$lbf/in^{3/2}$
K_0	reference value of K (see Section 3)	$N/m^{3/2}$	$lbf/in^{3/2}$
K_{Ic}	plane strain fracture toughness in Mode I	$N/m^{3/2}$	$lbf/in^{3/2}$
ΔK	stress intensity factor range	$N/m^{3/2}$	$lbfin/^{3/2}$
ΔK_{eff}	effective stress intensity factor range defined as $K_{max} - K_{cl}$	$N/m^{3/2}$	$lbf/in^{3/2}$
ΔK_p	stress intensity factor range corrected for plasticity	$N/m^{3/2}$	$lbf/in^{3/2}$
ΔK_{TH}	threshold value of ΔK	$N/m^{3/2}$	$lbf/in^{3/2}$
M	bending moment per unit thickness	(N m)/m	(lbf in)/in
N	number of load cycles		
n	characteristic of slope of crack growth rate curve		
P	point load per unit thickness	N/m	lbf/in
R	stress ratio, S_{min}/S_{max}		
r	polar co-ordinate of stress in component	m	in
r'	hole radius in infinite plate	m	in
r_p	plasticity correction term	m	in
S	nominal gross section stress in crack-free part of component	N/m^2	lbf/in^2
S_c	critical stress at failure	N/m^2	lbf/in^2
t	thickness of component in region of crack tip	m	in
a	geometrical factor in stress intensity factor		
a_1	geometrical factor in stress intensity factor corrected for plasticity		
σ_y	yield (or 0.2 per cent proof) stress of material	N/m^2	lbf/in^2
σ_r	stress along axis of polar co-ordinate r	N/m^2	lbf/in^2
$\sigma_{r\theta}$	shear stress in $r\theta$ plane	N/m^2	lbf/in^2

σ_θ	stress perpendicular to polar co-ordinate r	N/m^2	lbf/in^2
θ	angle of polar co-ordinate r from x-axis	rad	rad
v	Poisson's ratio		

Suffixes

I, II and III	indicate crack opening modes (see Sketch 3.1)
max, *min*	indicate maximum and minimum values of stress or stress intensity factor in loading cycle
f, *i*	indicate final and initial crack length (see Equation (4.10))

SI and British units are shown but any coherent system of units may be used.

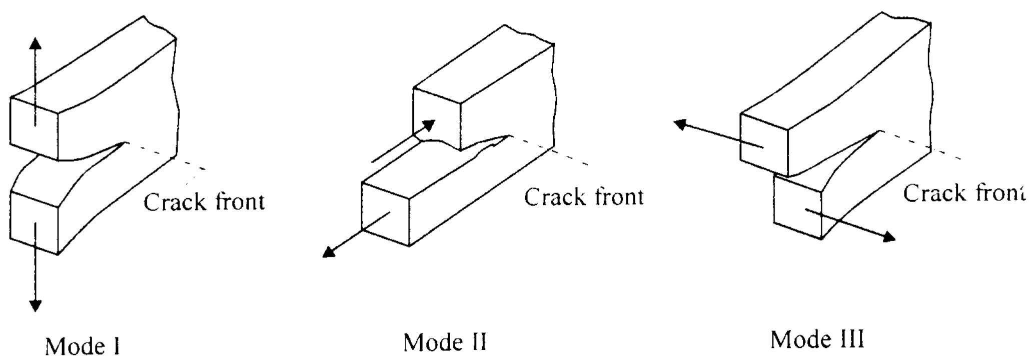
3. ANALYSIS OF CRACKED BODIES USING LINEAR ELASTIC FRACTURE MECHANICS

In analysing the stresses close to a crack tip, the concept of using elastic stress concentration factors breaks down. This is because the crack tip radius is assumed to approach zero and hence the stress concentration factor tends to approach infinity. The linear elastic fracture mechanics approach avoids this difficulty by identifying certain characteristics of the stress field that surrounds the crack tip and analysing them, rather than attempting analysis in terms of the 'infinite' stress at the crack tip. Allowances for a limited amount of plastic flow at the crack tip may be made (see Section 3.1.1).

Cracks grow in a variety of ways and the stress fields that develop around the crack tips may be divided into three basic modes. The modes are listed here and illustrated in Sketch 3.1. The Sketch shows small sections, in the vicinity of the crack, of much larger components.

Mode I Tension, normal to the faces of the crack (opening mode)
Mode II Shear, normal to the crack front in the plane of the crack (edge sliding mode).
Mode III Shear, parallel to the crack front (tearing mode).

Mode I is the mode of crack opening that occurs most commonly and this Item is concerned with the analysis of the Mode I opening only. Data obtained under one particular mode cannot be applied to any other mode.



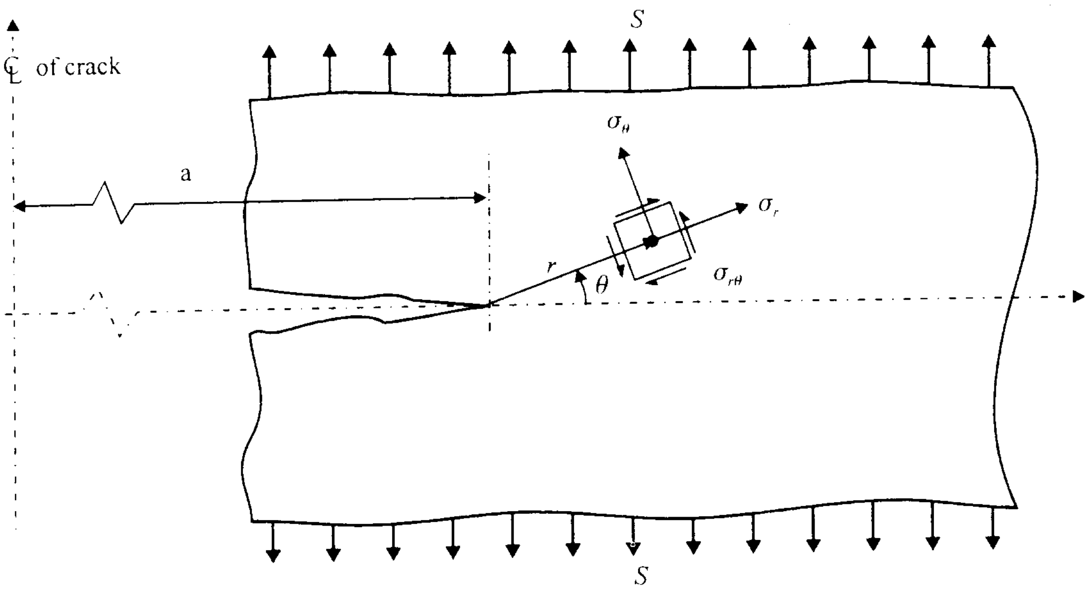
Sketch 3.1 Illustrating the three basic modes of crack growth

A two-dimensional loading model is used to describe the stress field but this model may be used to approximate to many three-dimensional loading cases that are met in practice. Assuming the material behaves in a linear elastic manner, the stresses σ_r, σ_θ, and $\sigma_{r\theta}$, developed in an element in the immediate neighbourhood $(r \ll a)$ of the tip of a crack in any loaded body, can be expressed in terms of the polar co-ordinates r, θ (see Sketch 3.2), in the form (see References 3 and 10)

$$\sigma_r, \sigma_\theta, \sigma_{r\theta} = \frac{K}{(2\pi r)^{1/2}} f(\theta) \quad \text{for } r \ll a, \tag{3.1}$$

where $f(\theta)$ is a different function for each of the three stresses. The terms $(2\pi r)^{1/2}$ and $f(\theta)$ map the stress field in the vicinity of the crack tip and they are the same for all cracks under any external loading system that causes crack opening of the same mode. They are linked by a scale factor, the so-called crack-tip stress-field-intensity factor, denoted K; the similarity of the mapping terms and the use of the scale factor are the essence of linear elastic fracture mechanics. The scale factor, K, is a function of the nature and magnitude of the applied stress levels, the crack size and other geometrical features, and is commonly referred to as the stress intensity factor.*

The form of Equation (3.1) is applied to all the modes of crack opening.



Sketch 3.2 Illustrating co-ordinate system and notation used in crack tip analysis

The geometry of a structure and the nature of applied loads determine the intensity of the stress field in the vicinity of the crack tip and hence the value of K, but do not affect the form of Equation (3.1). **The stress intensity factor, *K*, is a combination of various factors as described below and, where *K* is the same for different cracks even though the applied stress levels and geometries may be different, the stress fields very close to the crack tips will be identical.** Experimental data have shown that crack growth and residual strength data can be correlated better with K than other parameters.

* In some solutions found in the literature the stress intensity factor is denoted by k where $k = K/(\pi)^{1/2}$ (for example see References 14 and 18).

The value of K depends upon the nominal applied stress, which is usually taken as the nominal gross section stress, S, if the component were crack free, the crack length, a, and a term α that accounts for the effect of different configurations of the crack and its surrounding boundaries. It is usually expressed by*

$$K = S(\pi a)^{1/2}\alpha. \tag{3.2}$$

The term α is non-dimensional and is usually expressed as a ratio of crack length to any convenient local characteristic dimension in the plane of the component. Methods for finding solutions for α are described in Section 3.1. For the simple case of a crack in an infinite sheet or plate subjected to a uniform stress, S, remote from the crack, $\alpha = 1.0$ and K becomes $S(\pi a)^{1/2}$.

In some cases K may have an alternative form to that of Equation (3.2) and need not necessarily be expressed in terms of gross stress. For instance in cases where opposing loads P are applied at points close to the plane of the crack K can be expressed as

$$K = \left[\frac{P}{(\pi a)^{1/2}}\right]\alpha, \tag{3.3}$$

where P is the load per unit thickness. Note that for this loading configuration the value of α will be different from that in Equation (3.2) even for the same geometry.

Equations (3.2) and (3.3) may be expressed in terms of normalised stress intensity factors (see Reference 23) thus

$$K = K_0\alpha, \tag{3.4}$$

where K_0 is the reference value of stress intensity appropriate to the type of loading considered. In simple cases, such as the remotely loaded plate in tension, it is convenient to use

$$K_0 = S(\pi a)^{1/2}, \tag{3.5}$$

so that Equations (3.4) and (3.2) are identical and hence, for a given ratio of crack length to plate width, the value of α is the same for both formulations. In more complex cases, such as the in-plane bending of a plate of finite width $2b$ with central crack of length 2a, it may be convenient to express K_0 as (Reference 24)

$$K_0 = \frac{3Ma}{4b^3}(\pi a)^{1/2}, \tag{3.6}$$

where M is bending moment per unit thickness. By comparison with Equation (3.2) this implies that $S = 3Ma/4b^3$ which is not the conventional definition, but is the stress in the uncracked body at a distance $\pm a/2$ from the central axis. Note that the value of α depends on the value of S that is chosen, for example, $S = 3M/2b^2$ for the outer fibre stress might have been chosen, instead of the value $S = 3Ma/4b^3$ used in Equation (3.6).

The final value, K, must be independent of the form of presentation used. The use of Equations (3.2), (3.3) and (3.4) will depend on the form of solutions available for K_0 . Care must be taken that the formula used

* In some presentations the term $(\pi)^{1/2}$ is incorporated into the geometrical factors so that Equation (3.2) may be expressed as $K = S(a)^{1/2}\alpha'$, where $\alpha' = (\pi)^{1/2}\alpha$ (see Reference 26).

and the definition of nominal stress are consistent with those used in the derivation of the selected value of α.

For any one material and environmental condition, test data relating crack propagation rate and residual strength to *K* may be generated from relatively simple specimens and applied to cracks in other bodies for which the *K* values are the same but result from different combinations of load and crack length. It may be necessary to consider the extent of plastic flow in the vicinity of the crack tip and this is discussed in Section 3.1.1.

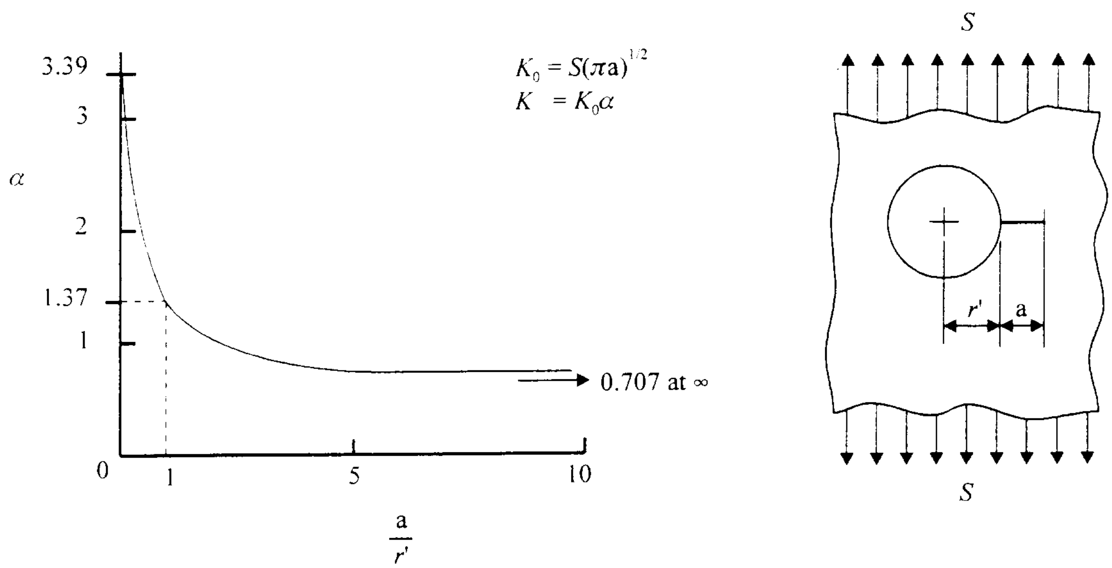
3.1 Solutions for *K* and α

There are several methods of obtaining *K* (see References 15, 17, 18, 23 and 24). The two most commonly used methods are as follows.

(i) Standard solutions for a wide variety of geometries and loading conditions are available from a number of sources (see References 3, 5, 9, 14, 17, 18, 25 and 30). Where a component has more than one type of loading, linear superposition may be used to obtain solutions for *K*. In cases where a crack is influenced by a combination of geometries, and a standard solution is not available, the compounding method may be used (see Reference 23 and Data Item No. 78036*).

(ii) Finite element analysis methods using special procedures are also possible where the discontinuity associated with a crack is incorporated.

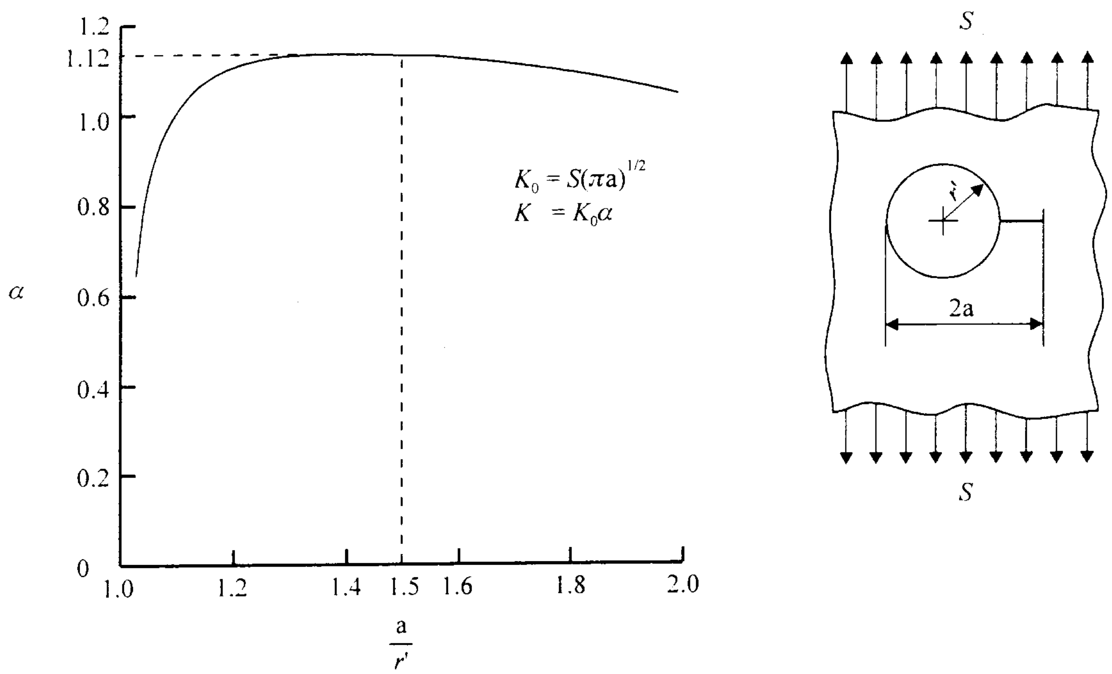
The particular definition of crack length, *a*, used in solutions for *K* and α is a matter of convenience, but care must be taken to ensure that the correct value of *a* is substituted into the appropriate expression. For example, Sketches 3.3 and 3.4 give alternative presentations of the solution for a single crack starting from a circular hole in an infinite sheet. Sketch 3.3 shows how α varies with the ratio of crack length to hole radius (a/r') where *a* is defined as the crack length from the edge of the hole and *r*' is the “convenient local characteristic dimension in the plane of the component” described in Section 3, earlier.



Sketch 3.3 Illustrating a definition of crack length that does not include the local geometry

* Data Item No. 78036 “The compounding method of estimating stress intensity factors for cracks in complex configurations using solutions from simple configurations”.

An alternative presentation of the same case, but where the hole is considered as part of the crack length, $2a$, is shown in Sketch 3.4. Note that for the two presentations K_0 is different according to the definition of a.



Sketch 3.4 Illustrating a definition of crack length that includes the local geometry

To show that the two presentations give identical values for K, the following example is given.

(a) *Solution for K from Sketch 3.3*

Assuming that the crack length is $a = r'$, then from Sketch 3.3 for $a/r' = 1.0$, $\alpha = 1.37$. Substituting into Equation (3.2) the values of α and a, the solution for K is given by

$$K = 1.37S(\pi r')^{1/2}. \qquad (3.7)$$

(b) *Solution for K from Sketch 3.4*

In this case the crack length is $a = 1.5r'$, and from Sketch 3.4 for $a/r' = 1.5$, $\alpha = 1.12$. Substituting into Equation (3.2) the values of α and a, the solution for K is given by

$$K = 1.12S(\pi 1.5r')^{1/2} = 1.37S(\pi r')^{1/2}. \qquad (3.8)$$

Hence, both methods give the same value of K.

A third solution for α would be obtained for this crack configuration if the crack length, a, were measured from the crack tip to the centre of the hole (see Reference 25).

3.1.1 Plasticity correction under steadily increasing loading

As noted in Section 3, a modification may be necessary to account for the effect of plastic flow that occurs in the vicinity of the crack tip. In such cases the size of the plastic zone is estimated and added to the real crack length to form an effective crack length, a_1, which is used in place of a.

The size of the plastic zone is described in terms of a plasticity correction*, r_p. For a crack in a component the effective crack length is therefore estimated as

$$a_1 = a + r_p . \tag{3.9}$$

Since α is a function of the crack length for a given geometry and loading, this factor should also be corrected for plasticity to become α_1 corresponding to a_1. The stress intensity factor corrected for plasticity is denoted by K_p and Equation (3.2) may then be re-written thus

$$K_p = S(\pi a_1)^{1/2}\alpha_1 . \tag{3.10}$$

When calculating the value of r_p it is necessary to consider the influence of component thickness on the stress state which is triaxial very close to the crack tip. In linear elastic fracture mechanics it is accepted that this triaxiality is less pronounced if the plastic zone surrounding the crack tip is large in relation to the thickness (usually in thin components). In practical applications the stress state at the crack tip can be described by one of the following three conditions,

(i) plane strain – a triaxial stress state where the plastic zone is very small compared with component thickness,

(ii) plane stress – a biaxial stress state where the plastic zone is large compared with component thickness and

(iii) somewhere between (i) and (ii) above.

These stress states are more fully described in Section 5.1.1.

An estimate of the extent of the plastic zone, r_p, for conditions of plane stress may be obtained for a particular value of K (see Reference 4) from

$$r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2 \text{ or } r_p = \frac{a}{2}\left(\frac{S}{\sigma_y}\right)^2\alpha^2 \tag{3.11}$$

and it has been estimated empirically that, for plane stress conditions to apply when using linear elastic fracture mechanics, $r_p > (t/2)$ so that, from Equation (3.11),

$$t < \frac{1}{\pi}\left(\frac{K}{\sigma_y}\right)^2 \approx 0.32\left(\frac{K}{\sigma_y}\right)^2 . \tag{3.12}$$

In the condition of plane strain the shape of the plastic zone is more complex but it is small and is taken to be about one third of the plane stress plastic zone size (see Reference 4) so that

$$r_p = \frac{1}{6\pi}\left(\frac{K}{\sigma_y}\right)^2 . \tag{3.13}$$

* The quantity r_p is commonly known as the radius of the plastic zone from analysis of the Mode III crack opening case in which the plastic zone for small scale yielding is circular (see Reference 4). For Mode I crack opening the plastic zone is not circular but r_p is retained as a dimensional correction.

For plane strain conditions to apply when using linear elastic fracture mechanics, it has been estimated empirically that $r_p < (t/50)$ so that, from Equation (3.13),

$$t > \frac{50}{6\pi}\left(\frac{K}{\sigma_y}\right)^2 \approx 2.5\left(\frac{K}{\sigma_y}\right)^2 . \qquad (3.14)$$

When the stress situation is between plane stress and plane strain the plasticity correction employed should be chosen so as to ensure that the more conservative answer is obtained. Although r_p is a function of K, and hence of K_p, it is not implied that the correction for a_1 in Equation (3.10) should be re-estimated by iteration.

4. USE OF FRACTURE MECHANICS TO ESTIMATE FATIGUE CRACK PROPAGATION RATES

In fatigue loading situations the process that eventually leads to failure of components can be conveniently divided into three successive stages, although the transition from one stage to the next is a continuous process and boundaries cannot be precisely defined. The three stages are as follows.

(i) Crystallographic slip and nucleation of cracks occurring on the atomic scale, of the order 10^{-9} m (3.9×10^{-8} in) in length or less.

(ii) Micro-growth of cracks ranging in length from 10^{-9} m (3.9×10^{-8} in) to about 10^{-4} m (3.9×10^{-3} in). The final stages of this phase may be just visible.

(iii) Macro-growth commencing in cracks of about 10^{-4} m (3.9×10^{-3} in) in length and continuing up to failure. By the time this stage is reached the cracks will be sufficiently large to grow as though the material were a homogeneous continuum.

References 7 and 22 give detailed descriptions of the above stages. Where the crack size is of the order of stages (i) and (ii) (often referred to as crack initiation), and where those stages take up a major part of the life, then estimates of the fatigue life of components can be obtained using material or component *S–N* data provided in the Data Items. However, if a component has a pre-existing crack of the order of the stage (iii) size then the early stages may be by-passed and the life will be much shorter. In this condition the use of linear elastic fracture mechanics is appropriate and the theory is applied primarily in two areas.

(a) Estimating the life of structures or components that contain cracks or crack-like defects.

(b) Establishing inspection intervals or permissible stress levels for cracked components. This will generally necessitate prediction of the growth of a crack under the load history appropriate to one inspection interval (or more) from an initial postulated crack. The length of the initial crack is generally taken to be the maximum that might be missed by the inspection method to be used for the component concerned. Crack growth prediction over one inspection interval, or more, is made to check that the crack will not grow within the inspection interval to such a length that the residual strength becomes inadequate. If the performance of the component is shown to be inadequate, it may be necessary to reduce the stress level in the component or to consider shorter inspection intervals in service.

The Data Items on crack propagation (referred to in Section 4.2) relate to the growth of visible macro-cracks, Stage (iii) of the fatigue process. However, for short cracks (that is, less than 1 – 2 mm (0.039 – 0.079 in) in many metallic materials) special data should be used, see Section 4.2.2.

4.1 Application to Fatigue Crack Propagation

Under constant amplitude fatigue loading the crack propagation rate of a given crack depends primarily on the range of stresses in the fatigue cycle, $(S_{max} - S_{min})$, and on the crack length. It is also influenced by the stress ratio, $R = S_{min}/S_{max}$. In simple cases, the stress intensity factor concept allows account to be taken of the two major terms by means of the stress intensity factor range, ΔK, defined by

$$\Delta K = K_{max} - K_{min} . \tag{4.1}$$

In its most simple, but not necessarily rigorous form, ΔK may be expressed as

$$\Delta K = (S_{max} - S_{min})(\pi a)^{1/2} \alpha . \tag{4.2}$$

The value of K_{max} is always a function of S_{max} in the form of Equation (3.2). However, the value of K_{min} may be governed by crack closure effects, as discussed in Sections 4.1.3 and 4.2.2.

The effect of mean stress on fatigue crack propagation rate, whilst important, is usually less important than the value of ΔK. Allowances for mean stress can however be introduced in terms of any convenient parameter, such as the stress ratio, R, or the maximum stress intensity factor in the fatigue cycle, K_{max}, or the mean stress intensity factor in the fatigue cycle, K_{mean}.

4.1.1 Presentation and use of crack propagation data

Experimental fatigue crack growth data are usually obtained from tests on simple specimens and are normally presented in terms of fatigue crack propagation rates (da/dN), ΔK and variations in values of R. In cases where S_{min} is compressive the crack may close during the fatigue cycle and no clear convention for calculating ΔK has been established. In the presentation of data for such conditions one of two approaches generally will have been used.

(i) The full range of the stress cycle will have been used when calculating ΔK.

(ii) Only the tensile part of the cycle will have been considered, that is $\Delta K = K_{max}$, (as if $K_{min} = 0$).

If results of independently derived test data are to be used, it is vital, when calculating ΔK for the component being analysed, to use the same method as has been applied to the analysis and presentation of the test data. The method of presentation of crack propagation rate data is stated in ESDU Data Items that provide such data.

4.1.2 Plasticity correction under cyclic loading

The plasticity correction under cyclic loading is smaller than the comparable value under monotonic loading. The change in stress at the crack tip due to cyclic loading is twice the value of the yield stress. Equations (3.11) and (3.13) for determining the value of r_p under steadily increasing loading conditions are modified by replacing K by ΔK and replacing the yield stress value, σ_y, by $2\sigma_y$. Neglecting the possibility of crack closure therefore, for conditions of plane stress, Equation (3.11) becomes

$$r_p = \frac{1}{8\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2 , \tag{4.3}$$

and, for conditions of plane strain, Equation (3.13) becomes

$$r_p = \frac{1}{24\pi}\left(\frac{\Delta K}{\sigma_y}\right)^2. \tag{4.4}$$

The appropriate value of r_p may then be added to the crack length, a, to obtain the effective crack length, a_1, (as seen in Equation (3.9)) for cyclic loading situations so that Equation (4.2) can be expressed as

$$\Delta K_p = (S_{max} - S_{min})(\pi a_1)^{1/2}\alpha_1 \tag{4.5}$$

As stated in Section 3.1.1, α_1 is normally corrected for plasticity once only, that is without iteration.

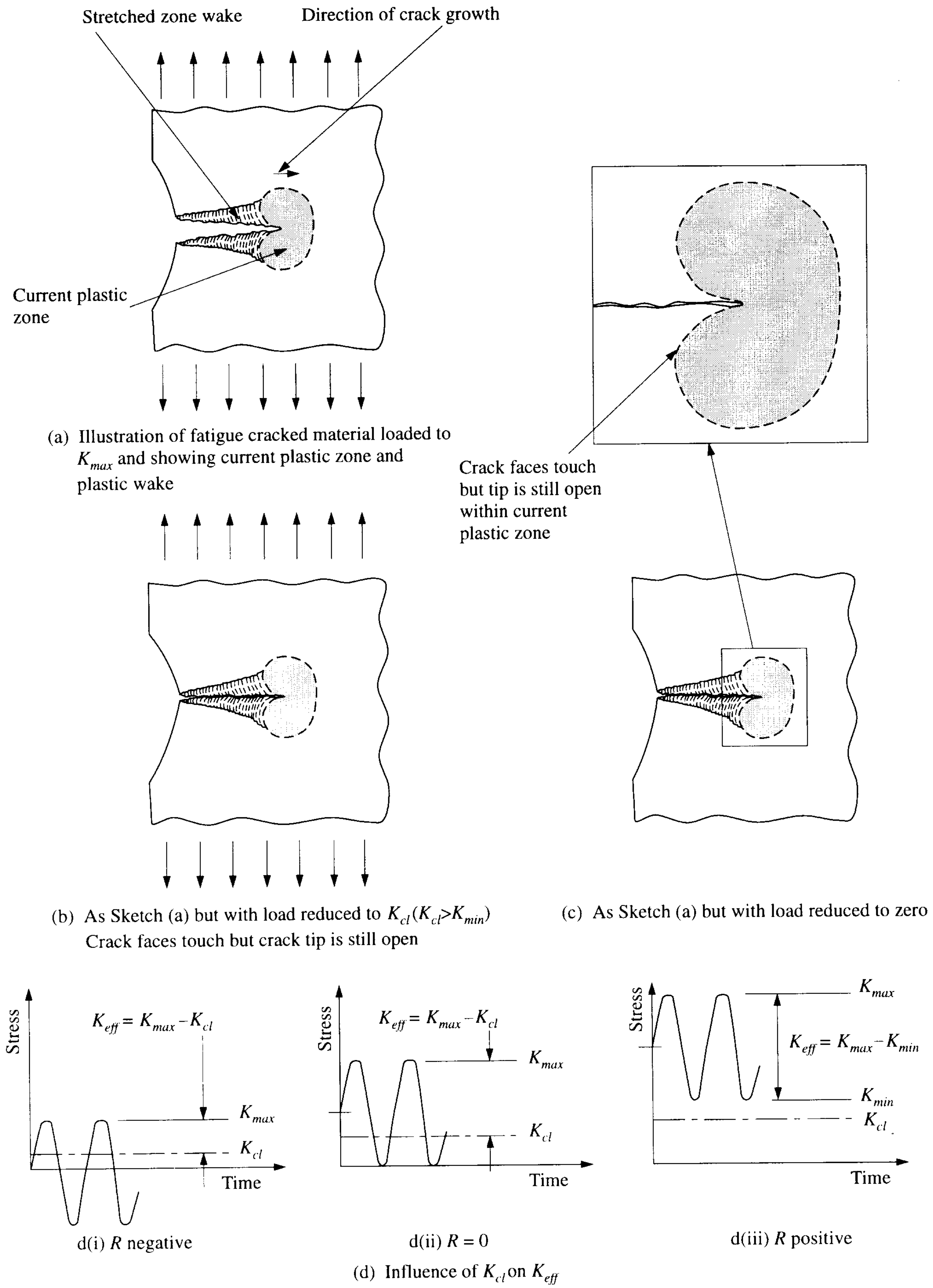
Note that in situations where an overload produces a plastic zone that is large in relation to the plastic zone due to cyclic loading, subsequent crack growth will be temporarily slowed. This phenomenon, primarily caused by compressive residual stresses, is commonly known as retardation.

4.1.3 Crack tip shielding

Fatigue cracks may experience crack tip shielding, whereby the effective range of crack tip intensity, ΔK_{eff}, is different from the nominal range, $\Delta K = K_{max} - K_{min}$. One mechanism of crack tip shielding is termed premature crack closure. A widely accepted model (see Reference 12) is that premature crack closure occurs as a result of crack tip plasticity. As a crack grows through and beyond the boundaries of the original crack tip plastic zone, a wake of plastically stretched material remains in the region through which the crack has propagated. This is shown in Sketches 4.1a to 4.1c.

During the unloading part of the fatigue cycle, premature closure of the crack occurs in this plastic wake, at K_{cl}, before zero nominal tensile stress has been reached whilst the actual tip of the crack within the active plastic zone remains slightly open, see Sketches 4.1b and 4.1c. Hence, ΔK_{eff} is less than the nominal ΔK, provided the calculated K_{min} is less than K_{cl}. This effect is even more significant when the lead cycle has a compressive portion, that is, when R is negative. When R is positive, premature crack closure effects are reduced and eventually disappear at high R (typically $R > 0.5$), see Sketch 4.1d(iii).

Additional causes of crack tip shielding, other than the plastic wake effect, may also occur. These include mismatch of crack face irregularities, lateral displacement of one crack surface relative to the other and debris in the crack. All of these may allow the crack faces to touch thereby keeping the tip of the crack slightly open, at and just above zero applied load. These and other models of crack tip shielding are reviewed in Reference 31.

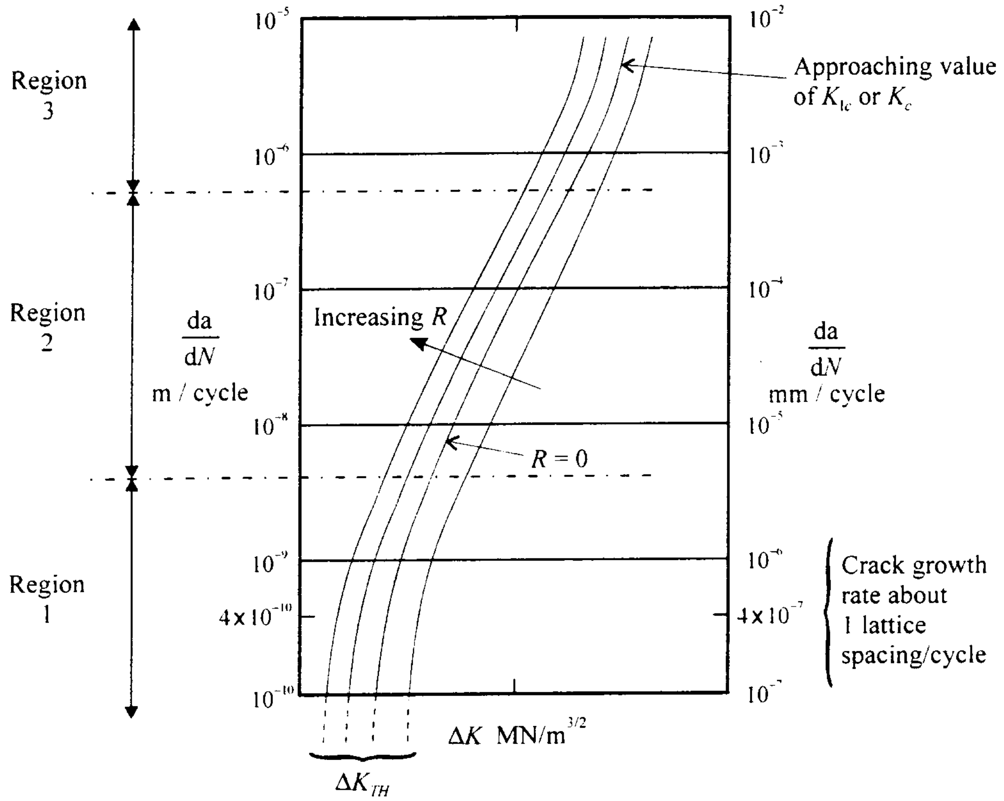


Sketch 4.1 Crack tip shielding, schematic of premature crack closure

4.2 Fatigue Crack Propagation Curves

For a wide variety of materials the fatigue crack propagation behaviour can be represented graphically by curves similar to those shown in Sketch 4.2. These curves may be described in general terms by a law of the form (Reference 1)

$$\frac{\mathrm{d}a}{\mathrm{d}N} = f(R, \Delta K)\,. \tag{4.6}$$



Sketch 4.2 Illustrating a typical set of crack growth rate curves

Many Data Items are available that present fatigue crack propagation rate data in a form similar to that shown in Sketch 4.2 for a wide range of materials; these may be located via the current ESDU Index.

The fatigue crack propagation rate curves may be divided up into three regions (as illustrated). Region 1 covers very slow crack growth rates (typically $<10^{-8}$ m/cycle (3.9 × 10^{-7} in/cycle)) where the curves approach a threshold value of stress intensity factor (that is, for no measurable crack growth). Region 2 covers the macro-crack growth rates (typically 10^{-8} to 10^{-6} m/cycle (3.9 × 10^{-7} to 3.9 × 10^{-5} in/cycle)) and this is the region where in typical practical cases (that is, components that contain pre-existing macro-cracks) much of the life will take place. Region 3 covers fast crack growth rates (typically $> 10^{-6}$ m/cycle (3.9 × 10^{-5} in/cycle)) in the region of instability and final failure. These three regions are discussed briefly below and in detail in Sections 4.2.1, 4.2.3 and 5, respectively.

In Region 1, at low values of ΔK, in the region of the threshold value of stress intensity, ΔK_{TH}, below which macro-cracks will not grow, the slopes of crack propagation rate curves are high and the effect of stress ratio, R, becomes increasingly more important. A crack growth rate of about 10^{-10} m/cycle (3.9 × 10^{-9} in/cycle) (which is less than 1 lattice spacing/cycle) is usually sufficiently small to define ΔK_{TH}, see Section 4.2.1.

For crack growth rates in Region 2 it has been found that over small ranges of ΔK Equation (4.6) can be represented by (see References 1 and 2)

$$\frac{da}{dN} = C(\Delta K)^n , \tag{4.7}$$

where the values of C and n are dependent upon the material properties and mean stress under consideration. They define the intercepts and slopes of the crack propagation rate curves, and over small ranges of da/dN and ΔK, within Region 2, their values remain approximately constant and Equation (4.7) is adequate. The effect of changes in the value of R on crack growth rates may therefore be accounted for by changes in the values of C and n within Region 2. Equation (4.7) however does not adequately describe the curves in Regions 1 and 3 and therefore when using the equation to predict crack growth life these limitations should be noted (see Section 4.2.3).

In Region 3, at high values of ΔK, the slopes of the crack propagation rate curves increase so that the value of $n \to \infty$. In some materials the magnitude of ΔK at high crack growth rates approaches a limiting value of ΔK such that

$$\frac{\Delta K}{(1 - R)} = K_{max} \to K_{Ic} \text{ or } K_c . \tag{4.8}$$

It should be noted that crack propagation rate curves do not necessarily follow in detail the form of Sketch 4.2 and the value of K at failure in some materials is not necessarily the value of K_{Ic} or K_c (see Section 5.1), particularly if plasticity becomes extensive, for example in the naturally aged aluminium alloys and in many structural steels.

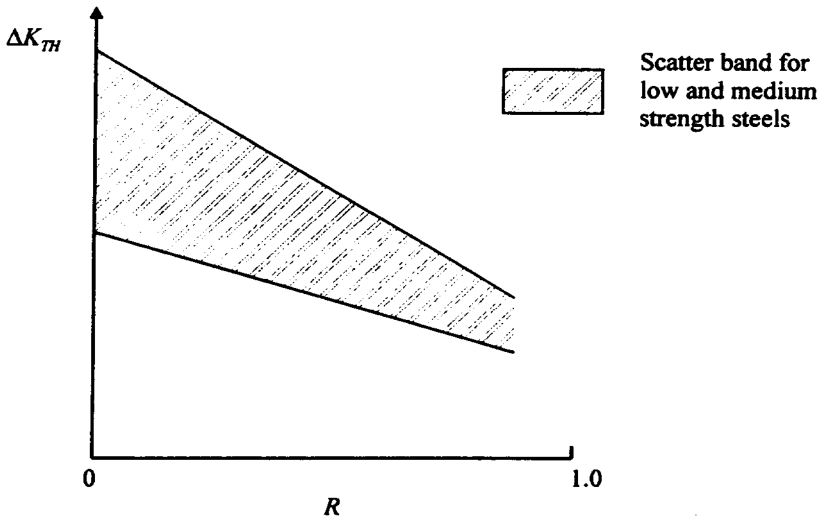
There have been many attempts made to derive laws that describe the complete crack propagation curves; many of these laws are discussed in Reference 21. One example that has been used in aerospace applications is the equation proposed in Reference 8. This expression takes into account the region of the curves as they near the value of K_c at high crack growth rates but does not adequately describe the low crack growth rate regime. It is expressed as

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1 - R)K_c - \Delta K} . \tag{4.9}$$

The integration of Equations (4.7) and (4.9) by analytical or graphical methods enables the designer to make an estimate of the crack growth life of a structure or component, see Section 4.2.3. The term “crack growth life” in this case is simply the number of load cycles needed to make a crack grow from an initial size (or length), a_0, until it reaches an unacceptable length, a_c, or an unacceptable crack growth rate is reached, or failure of the component occurs, whichever is the design criterion. Note that in some cases it may be necessary to check whether or not the fatigue cracked component is able to withstand a separate static design loading.

4.2.1 Threshold stress intensity factor, ΔK_{TH}

Flaws or crack-like defects in components will propagate under fatigue loading if the stress intensity factor is above the threshold stress intensity factor, ΔK_{TH}, for the particular material. If ΔK can be held below the value of ΔK_{TH} for the material then an effectively infinite fatigue life can be obtained despite the presence of cracks and flaws. In practice this tends to lead to very low design stresses even for quite small macro-cracks, but values of ΔK_{TH} are of interest when designing lightly stressed components that are subjected to large numbers of service loadings. In cases where components are crack free or contain flaws or cracks of the size defining Stages (i) and (ii) of Section 4 and are subjected to normal design stresses then the crack initiation stage may occupy many loading cycles. The overall fatigue life will then include both the initiation and propagation stages.



Sketch 4.3 Typical variation ΔK_{TH} with stress ratio, *R*

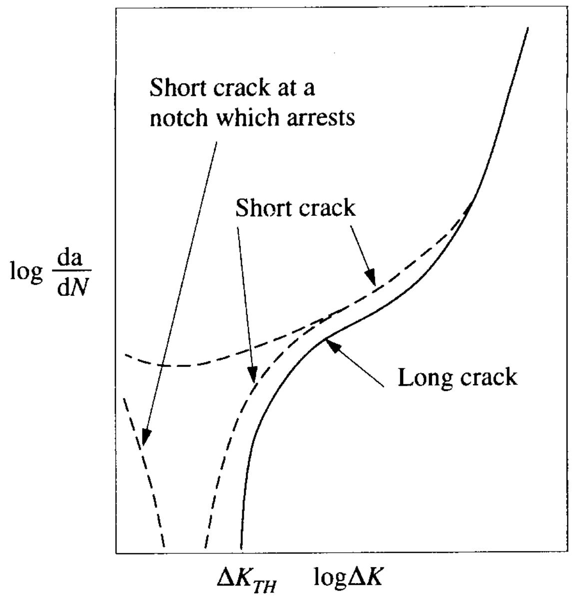
Values of ΔK_{TH} for materials appear to be very sensitive to variations in values of the stress ratio, *R*. They have also been shown to be sensitive to component stress history, corrosive environment and the mechanical and microstructural properties of the material. Some of these effects may be due to crack closure. As an example, the effect of *R* is important at slow crack propagation rates particularly when considering low and medium strength steels which in the normal crack propagation rate regime (Region 2 of Sketch 4.2) show little sensitivity to changes in the value of *R*. Sketch 4.3 typically shows how ΔK_{TH} varies with *R* for such steels.

4.2.2 The short crack effect

When employing linear elastic fracture mechanics to analyse the propagation of cracks of different lengths, significant differences in behaviour are observed between long cracks and short cracks. A short crack is described as one that is microstructurally small, that is, its length is close to that of the dominant microstructural dimension (grain size) and/or physically small, that is, not substantially larger than the plastic zone at the crack tip.

When crack propagation rates are plotted against the nominal ΔK, as determined by linear elastic fracture mechanics, the differences in behaviour between long and short cracks become apparent. Long cracks display a threshold value of stress intensity factor range, ΔK_{TH}, below which they do not propagate. However, short cracks will often appear to have no measurable threshold value of ΔK and can grow at values of ΔK below the long crack threshold value, ΔK_{TH}. Also the propagation rates of short cracks, as determined by linear elastic fracture mechanics, are often significantly higher than the propagation rates of long cracks at the same values of ΔK and *R*. The differences in crack propagation rates tend to decrease with increasing ΔK until both the long and short cracks have the same propagation rate. These differences in behaviour are illustrated in Sketch 4.4.

Crack tip shielding, and in particular the effect of premature crack closure, is a major factor in explaining the differences in crack propagation behaviour between long and short cracks. As already seen in Section 4.1.3, in a long crack premature crack closure can shield the crack tip from the full fatigue cycle. The effective stress intensity factor range, ΔK_{eff}, is then less than the nominal stress intensity factor range, ΔK .



Sketch 4.4 Comparison of the crack propagation rates of typical long and short cracks and for short cracks growing from a notch which arrests. All curves at a common value of stress ratio, *R*.

On the other hand whilst cracks are still within the initial plastic zone boundary, the wake of plastically deformed material may not be fully developed. Hence, short cracks are unlikely to experience premature crack closure. As a short crack grows, plastic wake effects will begin to develop more fully and its K_{cl} will usually increase, resulting in a decrease in the value of ΔK_{eff}, This reduction may negate, or even outweigh, the natural increase in ΔK_{eff} due to the increase in crack length. This effect is illustrated in Sketch 4.4. In the extreme case of a microstructurally small crack developed at a notch, there may be a region of non-propagation.

As has been explained, the assumptions of linear elastic fracture mechanics are not strictly valid for short cracks. However, it is convenient to use the same method of analysis for both long and short cracks, particularly in a damage tolerance analysis where a short crack is assumed to be the initial defect. It is therefore necessary to allow for the short crack effect since, if only long crack data are used, inaccurate and non-conservative endurances will be obtained.

Although it is extremely difficult to calculate *K* accurately for short cracks it is necessary to allow for short crack behaviour. A geometric shape function, α, must be assumed for the crack (see References 24 and 30). As the crack grows it may change shape requiring a change in the shape function. If the shape function is taken to be constant, large errors in *K* can result. The calculated stress intensity values, *K*, are only valid for specific regions of the crack, for example, at the point of maximum depth in a semi-elliptical surface crack. Short cracks are very susceptible to local stress fields that may differ from the assumed stress field, for example, a short surface crack may experience a stress field resulting from surface treatment or machining that is very different from the nominal stress field.

Short cracks often propagate along favoured crystallographic planes and the orientation of these planes can change from one grain to another. Thus, short cracks will often propagate along a plane that is inclined to the plane normal to the loading (for uniaxial loading). In such a case the short crack will be subjected to mixed mode loading. This is particularly likely in aluminium-lithium alloys which are noted for their deviating and branching fatigue crack paths. Fatigue cracks in aluminium-lithium alloys also experience higher levels of premature crack closure than conventional aluminium alloys. It is difficult to measure the curvature of the crack tip and its orientation with respect to the plane normal to the loading. These factors are not usually fully allowed for in the calculation of ΔK and the procedures are usually more complex. In some cases a length is calculated based on the surface length of the crack and an assumed shape function.

Data for short cracks presented using linear elastic fracture mechanics may be located via the current ESDU Index. The data are accompanied by notes on their application.

4.2.3 Determination of fatigue crack growth life

In Region II, the analytical integration of Equations (4.7) and (4.9) over several discrete values of da/dN and ΔK and summation of the discrete values of N will lead to an estimate of the fatigue crack growth life. This integration procedure can only be readily performed analytically if the value of n is an integer greater than 2 and the shape function factor, α, is in the form of a polynomial, $f(a)$. For example the integration of Equation (4.7) for the case of an infinite width plate ($\alpha = 1.0$) yields, provided that $n \neq 2$, the solution

$$[N]_{N_i}^{N_f} = \frac{1}{C[(S_{max} - S_{min})\pi^{1/2}]^n} \left[\frac{a^{[1-(n/2)]}}{1-(n/2)} \right]_{a_i}^{a_f} . \qquad (4.10)$$

Other solutions to the integral for finite width components may be found in References 31 and 32.

If Equations (4.7) and (4.9) cannot be readily integrated analytically, graphical solutions using step by step summations of N over discrete ranges of da/dN and ΔK can be used. In yet more complex cases numerical integration procedures may be used such as a Runge-Kutta method (see Reference 32, for example).

If the fatigue loading is not of constant amplitude the analysis of the loading spectrum may be achieved using the methods described in Item No. 95006*. If constant amplitude fatigue crack propagation rate data are used to analyse variable amplitude loading data then, generally, an underestimate of fatigue crack growth life will be obtained. Also, if one or more tensile overloads occur during the fatigue loading cycle these may produce a retardation effect on the crack growth and again the fatigue crack growth life will be underestimated. If allowance is to be made for load sequence effects other models may have to be used similar to those proposed in Reference 11.

* Data Item No. 95006 in the Fatigue - Endurance Data Sub-series “Fatigue life estimation under variable amplitude loading using cumulative damage calculations”.

5. USE OF FRACTURE MECHANICS TO DESCRIBE STATIC FAILURE AND RESIDUAL STRENGTH

In order to estimate the strength of a cracked component subject to steadily increasing loads or the residual strength of a component in which a fatigue crack has grown, the designer must have data that are derived from material toughness testing. Sections 5.1 and 5.1.1 discuss the concepts and definitions of fracture toughness. Practical testing methods for obtaining fracture toughness are briefly outlined in Section 5.1.2.

5.1 Fracture Toughness

Material testing should be carried out in the environment of practical interest (for example, temperature, humidity, strain rate, *et cetera*). In such testing the level of stress intensity is increased (by increasing the load) on a suitable cracked specimen until the initial failure occurs. The range of behaviour extends from brittle unstable fracture, with no apparent slow crack growth, through to crack growth or tearing that occurs in a slow stable fashion until the specimen has separated. Within this range a variety of mechanical effects may play a part, for example transitions between biaxial and triaxial stress states, growth of the plastic zone and changes in the micro-mode of separation. The value of stress intensity at fracture is considered to be the critical value and is known as the fracture toughness. It is designated either K_{Ic} (for thick sections where it is regarded as a material property) or K_c (for thin sections where toughness depends on thickness and developed plasticity). The underlying physical processes by which materials fracture are not yet fully understood and detailed descriptions of crack growth are not attempted here. The stress states and their associated plastic zones are discussed in more detail in Section 5.1.2.

To enable the designer to handle practical situations simplified concepts have been developed. These are applicable to typical classes of fracture behaviour and are described as follows.

(i) Unstable brittle fracture (fracture toughness denoted by K_{Ic}).

(ii) Fast fracture accompanied by a small amount of plasticity (fracture toughness denoted by K_c).

(iii) Fracture preceded by slow stable crack growth (fracture toughness denoted by K_c and obtained by the *R*-curve method).

Concept (i) is normally encountered in thick sectioned components of relatively brittle materials, whereas (ii) and (iii) are most likely to be encountered in relatively ductile materials and in thin sectioned components. Concepts (ii) and (iii) may in fact describe the same phenomena although (ii) does not explicitly include stable crack growth prior to failure. The *R*-curve approach of (iii) is discussed in detail in Section 5.2. It presents curves (known as crack growth resistance curves) that indicate a change in material toughness as stable crack growth occurs.

Apart from mechanical conditions that affect fracture, various environmental aspects may have a strong influence on the material response ahead of a crack. For example, in some ferritic materials the toughness deteriorates dramatically under conditions of high strain or low temperature.

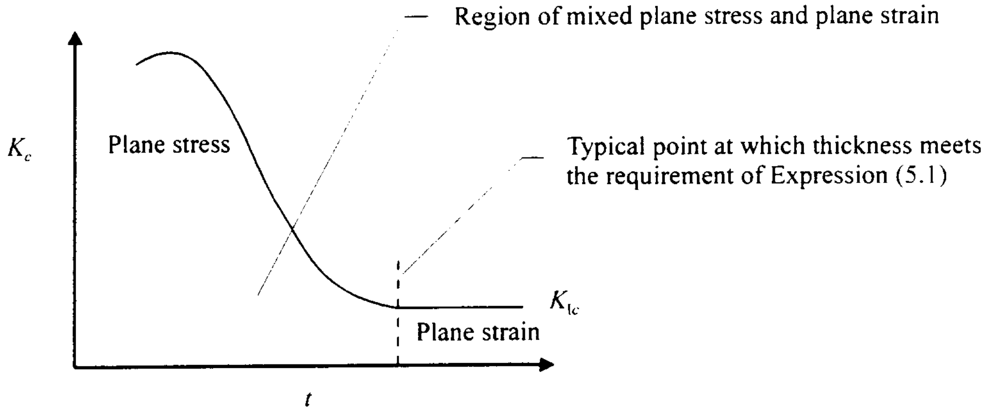
5.1.1 Effect of thickness on plane strain and plane stress conditions

If the crack tip plastic zone is small in relation to the component thickness, then plastic contraction in the through thickness direction is suppressed by the surrounding elastic material. Tensile stresses are set up in the thickness direction of the plastic zone so that the stress state is triaxial. This condition is called *Plane Strain*.

If the plastic zone is large compared with the component thickness, the triaxiality may be relaxed and the through thickness stresses normal to the plane of the component will be negligible. In this condition the size of plastic zone is substantially constant through the component thickness and the stress state is biaxial. The condition is called *Plane Stress*.

The stress conditions will influence the fracture toughness of materials. For example in thick components in which a condition of plane strain is applicable, the triaxial stress state suppresses the plastic zone size so that it is very small compared with the thickness and fracture toughness is low. Cracks develop normal to the faces (often termed flat fractures) through most of the component thickness but near the surfaces they may spread as shear fractures at 45° to the faces of the component (often termed shear lips). In thin sheet components, relaxation of the triaxial stresses allows a condition of plane stress to be reached giving an increase in the value of material fracture toughness. In plane stress conditions, cracks grow predominantly obliquely through the thickness at 45° to the faces of the sheet and the resultant fractures are denoted shear fractures. For intermediate thicknesses, fracture surfaces may be partly flat and become partly oblique depending on the value of thickness and toughness and, indeed, some details of all fracture behaviour depend on the type of material used.

Sketch 5.1 indicates a typical form of the relationship between fracture toughness and component thickness. The transition between plane strain and plane stress extends over a near ten-fold range of thickness (see Equations (3.12) and (3.14)).



Sketch 5.1 Typical form of relationship between fracture toughness and thickness

5.1.2 Practical testing to obtain K_{Ic} or K_c

For the purposes of conducting valid plane strain tests, the thickness of the specimen must be such that

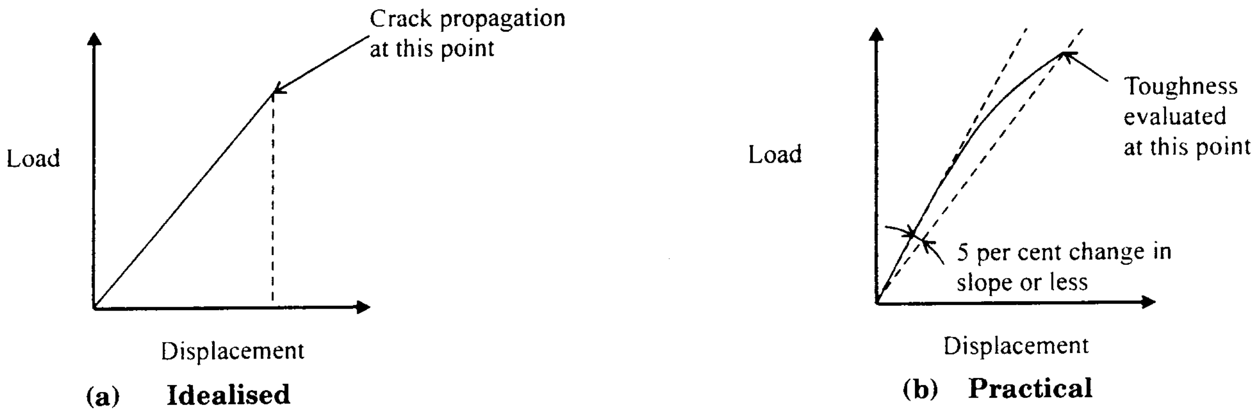
$$t > 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 . \qquad (5.1)$$

A state of plane strain (triaxial stress state) may still exist in a zone around the centre of a test specimen that is less thick than this recommended value. It is essential that determination of fracture toughness must be of the K_{Ic} type (discussed earlier) (Concept (i) of Section 5.1), and not of the K_c type if a conservative prediction of fracture behaviour is preferred.

There are standard laboratory test procedures for determining values of K_{Ic} for thick components (see References 33 and 34). Values of K_{Ic} for a selection of engineering steels are given in Data Item No. 83023*. In an idealised form the load-displacement diagram obtained from the test is as shown in Sketch 5.2(a). In

* Data Item No. 83023 in the Stress and Strength Sub-series, "Fracture toughness (K_{Ic}) values of some steels".

practice the load-displacement line is not perfectly straight because small amounts of plastic deformation occur at the crack tip and small amounts of stable crack growth also occur, but if the diagram obtained is reasonably linear (that is, there is less than 5 per cent change in the secant over the test) the fracture event can still be described by K_{Ic} , see Sketch 5.2(b).



Sketch 5.2 Local displacement curves obtained in tests for K_{Ic}

For toughness tests that do not satisfy the strict K_{Ic} criteria described above and that correspond to the conditions of Concept (ii) in Section 5.1, the point at which the crack propagates (unstable growth) is determined by the best method available and the material fracture toughness obtained is denoted K_c. Often a wide sheet is used for such a test and although there may be some stable crack growth this is neglected when determining fracture toughness. The value of K_c obtained from such a test is not strictly a material property, as is K_{Ic} , since it depends on material thickness and the amount of crack growth. Its value however will be conservative if it is used as a design criterion for a much larger sheet or panel (for example an aircraft wing panel).

If the load-displacement diagram shown in Sketch 5.2(b) is more non-linear than indicated and this non-linearity is due to slow stable crack growth at stress levels that are still predominantly elastic, as described in Concept (iii) of Section 5.1, then the case can be treated by the *R*-curve method which is described in Section 5.2. Nevertheless, if the non-linearity is due to the effects of plasticity, then analysis by linear elastic fracture mechanics may be inadequate. Methods of dealing with extensive plasticity have been proposed, namely crack opening displacement (COD) and *J*-contour integral, but are not further discussed here (see References 27 and 34).

The whole of the foregoing can be expressed in terms of a balance of energy rates. This concept is not described here, but the stress intensity factor *K*, and the potential energy release rate, *G*, are related by the equation

$$K^2 = HG , \tag{5.2}$$

where *H* is an effective modulus of elasticity such that, in the case of Mode I,

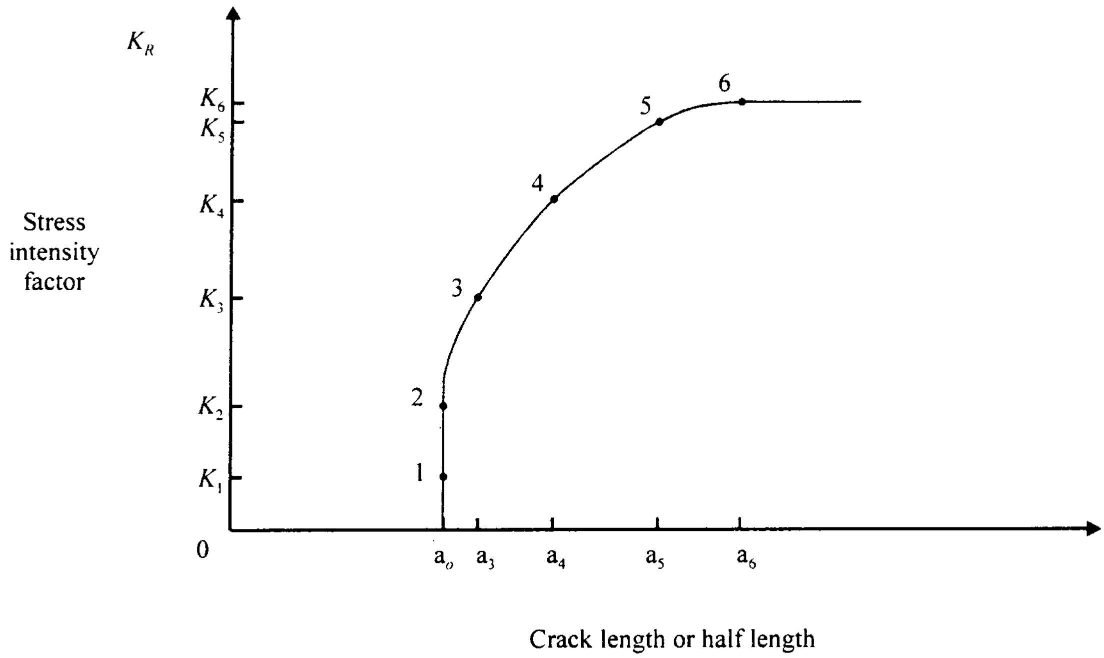
$H = E$ for plane stress

and $H = E/(1 - \nu^2)$ for plane strain.

Such an expression provides an insight into the physical meaning of fracture toughness and *K*, since the units of *G* are energy per unit area. This is equivalent to a force per unit thickness, as a result of which *G* is often referred to as the “crack driving force”.

5.2 Crack Growth Resistance Curves (*R*-curves)

As discussed in Sections 5.1 and 5.1.2 for materials where fracture is preceded by slow stable crack growth, fracture toughness (K_c) may be obtained by the use of crack growth resistance curves (commonly called *R*-curves). These curves are a continuous record of toughness development in terms of crack growth resistance, denoted K_R, plotted against crack extension under continuously increasing values of stress intensity factor, K (see Sketch 5.3). The *R*-curves characterise the resistance to fracture of material during incremental slow-stable crack extension as a result of the growth of the plastic zone as the crack extends. If a component, containing a crack or crack-like defect, and experiencing some plasticity in the vicinity of the crack, is loaded by increments the crack will extend and stop after each increase in load. This condition is defined as slow-stable crack growth. In this condition the value of material resistance, K_R, is equal to the applied value, K, at any given applied stress, S. As the applied stress increases the crack extends and measurements of crack extension are made at each value of stress. Values of K are calculated using either Equation (3.2) or (3.10), whichever is appropriate (note that plasticity corrections at the crack tip should be made in accordance with Section 3.1.1).



Sketch 5.3 Typical *R* curve

Sketch 5.3 may also be presented in terms of G, the potential energy release rate and crack length, a. From Sketch 5.3 it can be seen that, for the *R*-curve depicted, initially there is no visible crack growth as the stress increases so that the stress intensity factor for the crack is initially K_1 and then becomes K_2. Further increases in stress make the crack grow slowly and in a stable manner and the stress intensity factor increases from K_2 through to K_4. Eventually the crack starts to grow more rapidly with only small increases in applied stress and the stress intensity factor increases from K_4 to K_5 (the start of unstable crack growth) until it reaches a point K_6 where the specimen or component fails without further increases in stress. The applied stress at the onset of unstable crack growth is normally used to calculate the critical stress intensity factor, K_c, namely the fracture toughness in plane stress for a component of the same thickness. If the test specimen or component is a very wide plate or sheet the value of the K_c obtained will be as referred to in Section 5.2.1.

A recommended practice for determining *R*-curves is given in Reference 37. Crack resistance curves are usually determined for specimens of standard proportions but variable size, as recommended in the appropriate testing standard. However, the size of the specimen will affect the extent of the curve that can be obtained. Thus for ductile materials with high toughness it is not possible to use data obtained from small specimens to predict fracture instability behaviour in large structures. Data Item No. 85031* gives resistance curve data for aluminium alloys, titanium alloys and steels.

5.2.1 The determination of resistance curves for wide panel specimens

In order to provide basic data for the prediction of failure in large structures that experience slow stable crack growth at overall elastic conditions of stress, crack resistance curves must be developed using large specimens. However, the size of the specimen that can be tested will be influenced by the size of the available test apparatus. For example, the testing standard for a centre crack tension (CCT) test piece requires that the length of the test piece must be greater than 1.5 times the width in order for the test piece to be stressed uniformly. Thus test pieces up to 2 m (78.7 in) wide would have to have a length of 3 m (118.1 in), requiring a crosshead spacing on the test machine of approximately 4 m (157.5 in). Reducing the ratio of the specimen length to width would allow testing without the need for greater crosshead spacing and would have the added advantage of reducing the amount of test material required.

However, it is necessary to calibrate experimentally the normalised compliance specific to the non-standard specimen to be used. A valid test can be conducted when the geometric correction factor (and hence the stress intensity factor, K) and the compliance against normalised crack length calibration have been determined for any required length to width ratio of less than 1.5 (for CCT specimens).

It has been shown that for CCT specimens valid tests can be carried out for non-standard specimen geometries for a range of length to width ratios of between 2 and 0.5. Furthermore, it has been shown that resistance curve tests carried out on a range of panel sizes showed good agreement (see Reference 36).

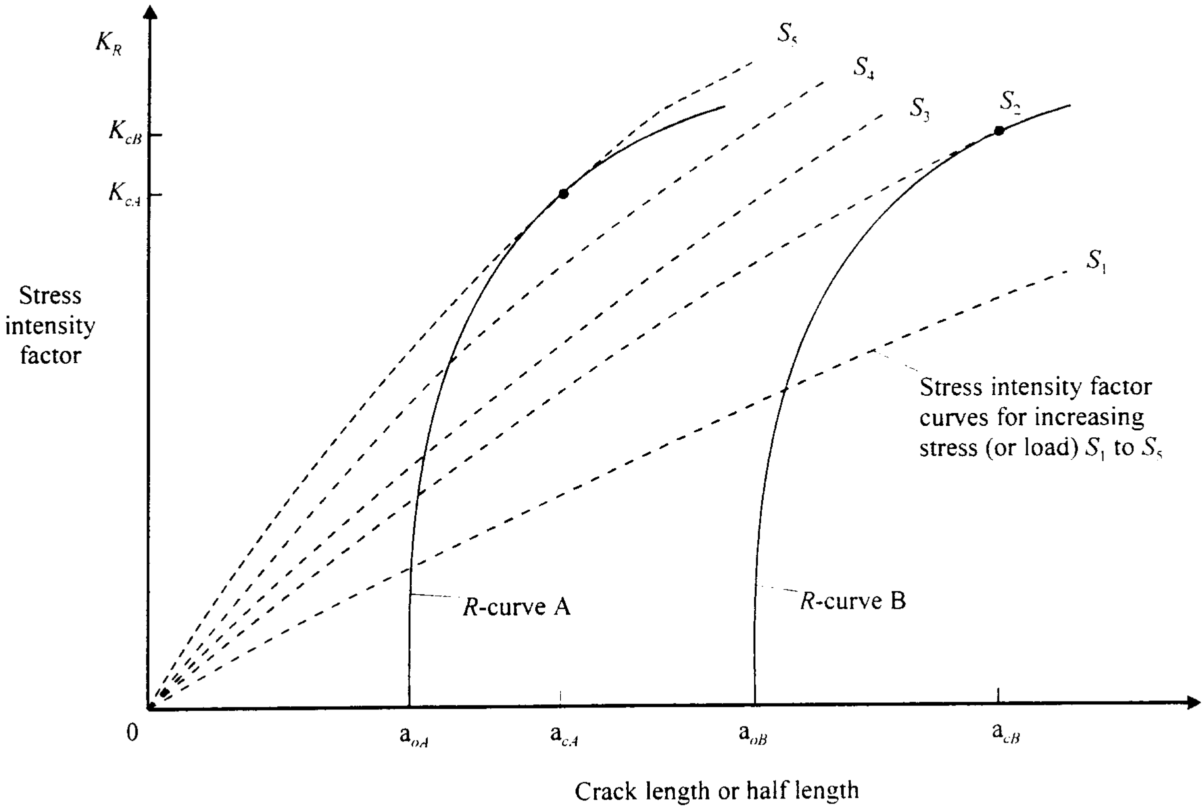
5.2.2 Use of *R*-curves

To predict the value of K_c, stress intensity factor curves for successively increased values of applied stress are generated (Sketch 5.4). These curves are obtained by calculating values of K for a given geometry, as a function of crack length for constant values of stress using the appropriate equations (either Equation (3.2) or (3.10)). The stress intensity factor curve that is tangential to the R-curve defines the critical stress intensity factor at the onset of unstable crack growth, for a critical crack length of a_c. The critical crack length prior to application of load is given by a, and is obtained empirically from the R-curve. Sketch 5.4 presents stress intensity factor curves S_1 to S_5 superimposed over R-curves for two different values of initial crack length, a_{0A} and a_{0B} . The curve S_5 is tangential to R-curve A and the point of tangency defines the fracture toughness, K_{cA}. Similarly the curve S_2 is tangential to R-curve B and defines the fracture toughness, K_{cB}.

The R-curve is dependent upon specimen size, temperature and strain rate but its shape is independent of initial crack length. Nevertheless, as seen in Sketch 5.4, the point of tangency of the stress intensity factor curves, used to predict instability, is a function of initial crack length and hence unstable crack growth may occur at different values of K_c (compare curves A and B of Sketch 5.4). See Data Item No. 85031* for a more detailed explanation, worked examples and R-curve data.

One of the uses of the R-curve in design is to find a_0; the method is to plot the stress intensity factor for the limit load in the component under consideration on the K_R diagram, for example S_5 on Sketch 5.4, then to move the R-curve along the axis of crack length, a, until a point of tangency to the stress intensity curves is achieved. The critical initial crack length, a_{0A} in this case, can then be read off the diagram.

* Data Item No. 85031 in the Fatigue - Fracture Mechanics Sub-series “Crack resistance curves”.



Sketch 5.4 Illustrating the use of *R*-curves

5.3 Residual Strength and Critical Crack Size

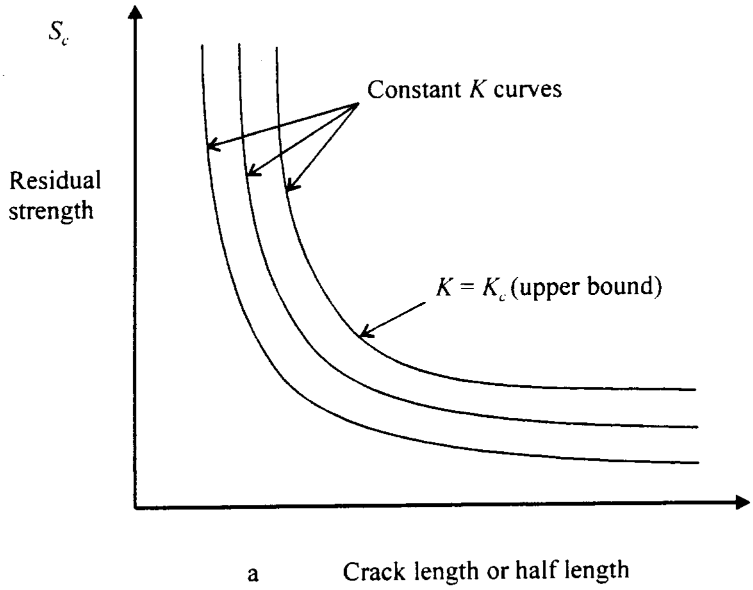
In Section 5.1, it is implied that fracture will occur when the stresses and strains reach a critical state, described by the material fracture toughness, in either plane strain or plane stress. The residual strength of a cracked body is a function of toughness, crack size and component configuration (which also influences the geometric factor, α). Equation (5.3) is based on the assumption that the static strength of the component is reached when the stress intensity factor exceeds the fracture toughness. Note that components may fail under static loading in those cases where, for instance, the net section stress exceeds the ultimate strength prior to the stress intensity factor exceeding the fracture toughness. The treatment of the determination of residual strength is discussed more fully in Reference 12. Expressed in terms of the applied stress, where the theory of linear elastic fracture mechanics applies strictly, the residual strength is given by the rearrangement of Equation (3.2),

$$S_c = \frac{K_c}{(\pi a)^{1/2}\,\alpha} \tag{5.3}$$

Note that S_c is the gross stress on the section on which the function α is defined, whereas residual strength implies a net section condition. In the case of plane strain, $K_c = K_{Ic}$. It is conservative to assume that $K_c = K_{Ic}$ if the detailed stress state is not known. The size of crack at this stress is called the "critical crack size" for the configuration and loading in question and is given by the alternative rearrangement of Equation (3.2). This is normally difficult to solve because α is a complicated function of crack length and component geometry. Nevertheless, if the value of α varied slowly with crack size, for example for a relatively small crack in a wide panel, an approximate value may be used and changes in the value of α as the crack grows may be ignored or allowed for by iteration. The rearranged equation is given by

$$a = \frac{1}{\pi}\left(\frac{K_c}{S_c\alpha}\right)^2 . \tag{5.4}$$

Using either Equation (5.3) or (5.4), curves may be plotted for selected constant values of K as a function of crack size and stress on a residual strength diagram (see Sketch 5.5). In those cases where the stress intensity factor solution is available, the residual strength for a given configuration and loading may be calculated knowing the fracture toughness of the material for a known crack size. A procedure for calculating the residual strength of centre cracked panels is given in Reference 12 and for stiffened panels in Reference 28.



Sketch 5.5 Typical plots of residual strength versus crack lengths for constant K

6. FURTHER READING ON BASIC PRINCIPLES

References 16, 19, 20 and 35 provide further reading on the fundamentals of fracture mechanics and Reference 28 gives some practical applications.

7. REFERENCES

The references given are recommended sources of information supplementary to that in this Item.

1. PARIS, P.C. ERDOGAN, F. — A critical analysis of crack propagation laws. *Trans. am. Soc. mech. Engrs, J. bas. Engng*, pp. 528-534, Vol. 85, Series D, No. 4, December 1963.

2. PARIS, P.C. — The fracture mechanics approach to fatigue. In: *Fatigue – an interdisciplinary approach*, pp. 107-132, Syracuse University Press, New York, USA, 1964.

3.	PARIS, P.C. SIH, G.C.	Stress analysis of cracks. In: *Fracture toughness and its applications*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 381, pp. 30-83, April 1965.
4.	McCLINTOCK, F.A. IRWIN, G.R.	Plasticity aspects of fracture mechanics. In: *Fracture toughness testing and its applications*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 381, pp. 84-113, April 1965.
5.	BROWN, W.F. SRAWLEY, J.E.	*Plain strain crack toughness testing of high strength metallic materials*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 410, December 1966.
6.	RICE, J.R.	Mechanics of crack tip deformation and extension by fatigue. In: *Fatigue crack propagation*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 415, pp. 247-311, June 1967.
7.	SCHIJVE, J.	Significance of fatigue cracks in micro-range and macro-range. In: *Fatigue crack propagation*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 415, pp. 415-459, June 1967.
8.	FORMAN, R.G. KEARNEY, V.E. ENGLE, R.M.	Numerical analysis of crack propagation in cyclic-loaded structures. *Trans. am. Soc. mech. Engrs, J. bas. Engng*, pp. 459-464, Vol. 89, Series D, No. 3, September 1967.
9.	BRADSHAW, F.J. ROOKE, D.P.	The application of fracture mechanics to material selection and structure design in aircraft. Royal Aircraft Establishment, Farnborough, UK. RAE TR 69122, June 1969.
10.	BARROIS, W.G.	Manual on fatigue of structures. Fundamental and physical aspects. Advisory Group Aerosp. Res. Dev., NATO, France, AGARD-MAN-70, June 1970.
11.	WILLENBORG, J.D. ENGLE, R.M. WOOD, H.A.	A crack growth retardation model using an effective stress concept. Air Force Flight Dynamics Lab., Wright-Patterson AFB, Ohio, USA. AFFDL-TM-FBR-71-1, January 1971.
12.	ELBER, W.	The significance of fatigue crack closure. In: *Damage tolerance in aircraft structures*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 486, pp. 230-243, 1971.
13.	FEDDERSEN, C.E.	Evaluation and prediction of the residual strength of center cracked tension panels. In: *Damage tolerance in aircraft structures*. American Society for Testing Materials, Philadelphia, USA, ASTM STP 486, pp. 50-78, 1971.
14.	BENTHEM, J.P. KOITER, W.T.	Asymptotic approximations to crack problems. In: *Mechanics of fracture. 1. Methods of analysis and solutions of crack problems*, pp. 131-178, Noordhoff International Publishing, Leyden, The Netherlands, 1973.
15.	CARTWRIGHT, D.J. ROOKE, D.P.	Methods of determining stress intensity factors. Royal Aircraft Establishment, Farnborough, UK, RAE TR 73031, May 1973.
16.	KNOTT, J.F.	*Fundamentals of fracture mechanics*. Butterworths, London, 1973.

17.	TADA, H. PARIS, P.C. IRWIN, G.	*The stress analysis of cracks handbook*. Del Research Corp., Hellertown, Pennsylvania, USA, 1973.
18.	SIH, G.C.	*Handbook of stress intensity factors for researchers and engineers.* Institute of Fracture and Solid Mechanics, Lehigh University, Bethlehem, Pennsylvania, USA, 1973.
19.	LIEBOWITZ, H.	Fracture mechanics of aircraft structures. Advisory Group Aerosp.Res. Dev., NATO, France, AGARD-AG-176, January 1974.
20.	BROEK, D.	*Elementary engineering fracture mechanics*. Noordhoff International Publishing, The Netherlands, 1974.
21.	HOEPPNER, D.W. KRUPP, W.E.	Prediction of component life by application of fatigue crack growth knowledge. *Engng Fract. Mech.*, Vol. 6, No. 1, pp. 47-70, 1974.
22.	FREUDENTHAL, A.M.	New aspects of fatigue and fracture mechanics. *Engng Fract. Mech.*, Vol. 6, No. 4, pp. 775-793, December 1974.
23.	ROOKE, D.P. CARTWRIGHT, D.J.	A method of compounding stress intensity factors for complex configurations. Royal Aircraft Establishment, Farnborough, UK, RAE TR 75063, June 1975.
24.	ROOKE, D.P. CARTWRIGHT, D.J.	*Compendium of stress intensity factors*. HMSO, London, UK, 1976.
25.	GRAY, T.G.F.	Convenient closed form stress intensity factors for common crack configurations. *Int. J. Fract.*, Vol. 13, No. 1, pp. 65-75, February 1977.
26.	BROEK, D.	Introduction to fracture mechanics. In: Fracture mechanics design methodology. Advisory Group Aerosp. Res. Dev, NATO, France, AGARD-LS-97, January 1979.
27.	LATZKO, D.G.H.	*Post yield fracture mechanics*. Applied Sciences Publications, London, UK, 1979.
28.	LIEBOWITZ, H.	Practical applications of fracture mechanics. Advisory Group Aerosp. Res. Dev., NATO, France, AGARD-AG-257, May 1980.
29.	GALLAGHER, J.	*Damage tolerant design handbook*. Metals and Ceramics Information Center, Battelle, Columbus Laboratories, Columbus, Ohio, USA, 1983.
30.	MURAKAMI, Y. AOKI, S.	*Stress intensity factors handbook*. Pergamon Press, Oxford, UK, 1988.
31.	RITCHIE, R.O.	Mechanisms of fatigue crack propagation in metals, ceramics and composites: role of crack tip shielding. *Mats. Sci. and Engng*, Vol. A103, No. 1, pp.15-28, August 1988.
32.	PRESS, W.H. *et al.*	*Numerical recipes. FORTRAN version*. Cambridge University Press, Cambridge, UK, 1989.
33.	ASTM	Standard test method for plane-strain fracture toughness of metallic materials. American Society for Testing and Materials, Philadelphia, USA, ASTM E399-90a, ASTM Standards Handbook, 1990.

34. BSI — Fracture mechanics toughness tests: Part 1: Method for determination of K_{Ic}, critical CTOD and critical J values of metallic materials. British Standards Institution, London, UK. BS 7448: Part 1: 1991.

35. BSI — Guidance on methods for assessing the acceptability of flaws in fusion welded structures. British Standards Institution, London, UK. PD 6493: 1991.

36. SUTTON, G.R. *et al.* — Determination of *R*-curves for short, wide centre cracked panels of aluminium alloy sheet. *Fatigue Fract. Engng Mater. Structures*, Vol. 14, No. 8, pp. 823-832, 1991.

37. ASTM — Standard practice for *R*-curve determination. American Society Testing and Materials, Philadelphia, USA. ASTM E561-92, ASTM Standards Handbook, 1992.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Fatigue Committee, which first met in 1955 and now has the following membership.

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The work on this Item was carried out in the Strength Analysis Group of the Engineering Sciences Data Unit under the supervision of Mr M.E. Grayley, Group Head. The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Miss A.B. Mew – Engineer.